Math 2S3, Test 1
Bradd Hart, Feb. 6, 2015

Please write complete answers to all of the questions in the test booklet provided. Partial credit may be given for your work. Unless otherwise noted, you need to justify your solutions in order to receive full credit. Please be sure to include your name and student number on all sheets of paper that you hand in.

1. (5 marks)

    (a) Explain what it means to say that $(V, +)$ is an abelian group.

    (b) Give an example of a group which is not abelian.

2. (5 marks)

    (a) Define what is meant by a linear transformation or linear map.

    (b) Show that if $f : V \to W$ is a linear map between two K-vector spaces V and W then $f(0) = 0$.

3. (5 marks) Show that for any linear map $f : V \to W$ between two K-vector spaces V and W, the image of f, $\mathrm{Im}(f)$, is a subspace of W.

4. (5 marks) Using any facts that were proved in the course, show that if V is a finite-dimensional K-vector space and W is a subspace of V then the dimension of W is less than or equal to the dimension of V.

5. (5 marks) Remember that an $n \times n$ matrix A over K is said to be symmetric if $A^t = A$ (A equals its transpose). What is the dimension of the space of all symmetric $n \times n$ K-matrices?