Assignment 3, Math 2S3
Due Feb. 25 in class

(1) Show that if A is an $n \times n$ matrix over K then
$$\{f \in K[x] : f(A) = 0\}$$
is an ideal. Conclude that the generator of this ideal is the minimal polynomial of A and it divides the characteristic polynomial of A.

(2) We say that an $n \times n$ matrix N is nilpotent if for some r, $N^r = 0$. What is the characteristic polynomial of a nilpotent matrix?

(3) Use the fact that every $n \times n$ matrix over C is similar to an upper triangular matrix to show that if A is an $n \times n$ matrix over C then $A = D + N$ where D is a diagonalizable matrix and N is a nilpotent matrix.

(4) Given an example of an $n \times n$ complex matrix with a single eigenvalue whose minimal polynomial has degree n and an example where the minimal polynomial has degree less than n.