Arts & Science 1D06 Quiz #2 October 7, 2015

Full Name: SOLUTIONS Student # : ____________

TA: Maddie

Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

[5 marks] (1.) Suppose you want to use Newton's Method to solve $\frac{1}{x} - a = 0$. Show that the formula for the $(n+1)$st value is $x_{n+1} = 2x_n - ax_n^2$.

$f(x) = \frac{1}{x} - a$

$f'(x) = \frac{-1}{x^2}$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

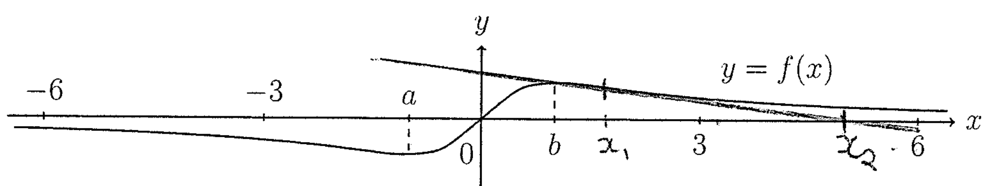
$x_{n+1} = x_n - \frac{\frac{1}{x_n} - a}{\frac{-1}{x_n^2}}$

$= x_n - (\frac{1}{x_n} - a)(-x_n^2)$

$= x_n - (-x_n + ax_n^2)$

$= x_n + x_n - ax_n^2 = 2x_n - ax_n^2$

[5 marks] (2.) Explain, with reference to the graph below, why applying Newton's Method does not work for finding the solution if the initial approximation is not in the interval (a, b).



For $x > b$, $f(x) > 0$ and $f'(x) < 0$. Thus $x_n - \frac{f(x_n)}{f'(x_n)}$ will be greater than x_n, ie $x_{n+1} > x_n$. For each iteration of Newton's Method, x_n will approach infinity.

Similarly for $x < a$, $f(x) < 0$ and $f'(x) < 0$. So $x_{n+1} < x_n$ for all $x < a$, ie $x_n \to -\infty$. Instead of getting closer to the root, choosing an initial value outside of (a, b) gets you further and further from the solution with each iteration.