Arts & Science 1D06 Quiz #2 - Solutions

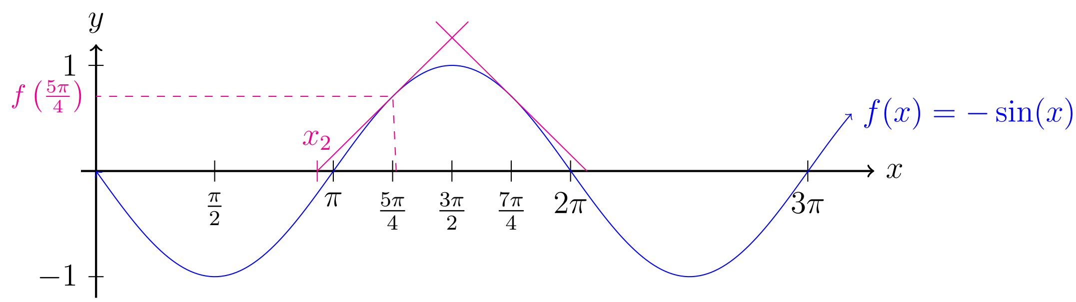
9 October 2015

Full Name:____ Student # :________________

TA:____

Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

[10 marks] (1) This question will explore how Newton's method can be both useful and temperamental using the function $f(x) = -\sin(x)$.



(a) *[4]* Imagine you are told to show that $f(x)$ has a root at $x = \pi$ using Newton's method. Explain why each of $\frac{\pi}{2}$, $\frac{3\pi}{2}$, and $\frac{7\pi}{4}$ would be pretty bad choices for your initial guess.

At both $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$, the tangent line is flat. This means the lines never intersect the $x-$axis, so it's impossible to apply Newton's method and find a better guess.

With $\frac{7\pi}{4}$, you will hit the $x-$axis, but at the "wrong" root. Looking at the pink tangent line at $\frac{7\pi}{4}$, it's clear that repeated applications of Newton's method will land us at the solution $x = 2\pi$.

(c) *[3]* Compute the numerical value of x_2. Is x_2 closer to π than x_1 is?
(Hint: $\frac{5\pi}{4} \approx 3.927$.)

The formula for Newton's method says:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}.$$

The derivative of f is $-\cos(x)$, so we have:

$$\begin{aligned} x_2 &= \frac{5\pi}{4} - \frac{-\sin\left(\frac{5\pi}{4}\right)}{-\cos\left(\frac{5\pi}{4}\right)} \\ &= \frac{5\pi}{4} + \tan\left(\frac{5\pi}{4}\right) \\ &= \frac{5\pi}{4} - 1 \\ &\approx 2.927 \end{aligned}$$

Yes, $x_2 \approx 2.927$ is closer to π than $x_1 \approx 3.927$ is.