Arts & Science 1D06 Quiz #2   8 October 2015
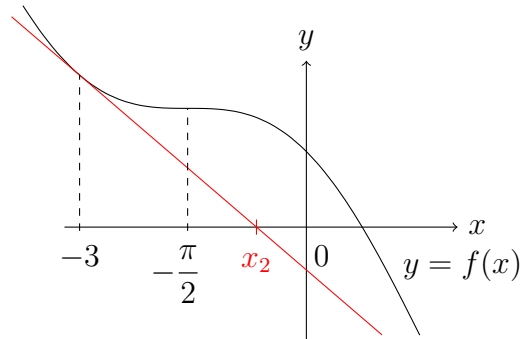
Full Name:___SOLUTIONS_____   Student # :_____

TA:___Max Lazar_____

Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

[10 marks]

(1) Consider the graph of the function $f(x) = \cos x - x$ below:

    (a) *[2]* Explain why choosing $x_1 = -\frac{\pi}{2}$ would be a bad choice for your initial guess to start using Newton's method.

If we pick $x_1 = -\pi/2$, then the tangent to $f(x_1)$ is a horizontal line, which will never cross the x-axis. As a result, we won't get an x_2 value. Another way to consider this is that $f'(x_1)$ would be 0 at $x_1 = -\pi/2$, so we can't get an x_2 value by using Newton's method.

    (b) *[4]* Given $x_1 = -3$ as your initial guess, sketch on the graph how you would find your next guess, x_2, using Newton's method.

    (c) *[4]* Given $x_1 = -3$, use Newton's method to compute x_2.

First we should compute $f'(x)$:
$f'(x) = -\sin x - 1$. Now we're ready to find x_2.

$$
\begin{aligned}
x_2 &= x_1 - \frac{f(x_n)}{f'(x_n)} \\
&= x_1 - \frac{\cos x_n - x_n}{-\sin x_n - 1} \\
&= -3 - \frac{\cos(-3) - (-3)}{-\sin(-3) - 1} \\
&= -3 - \frac{\cos 3 + 3}{\sin 3 - 1} \approx -0.660
\end{aligned}
$$