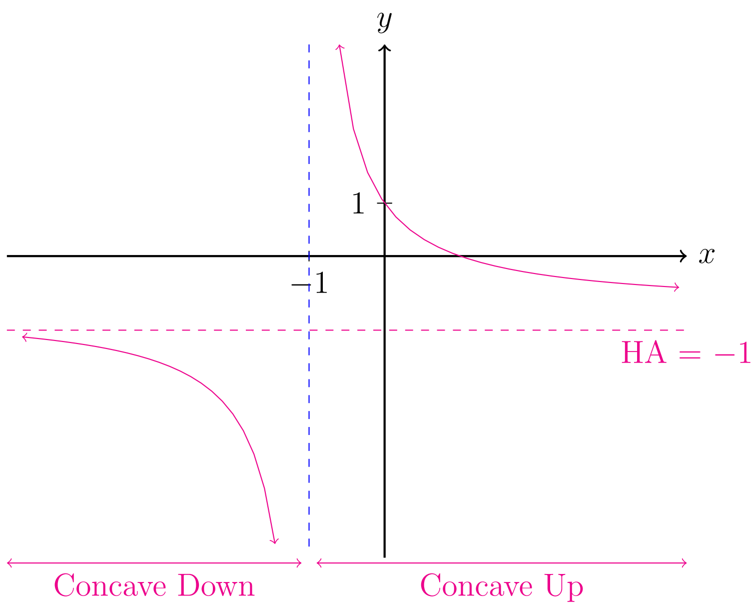
Arts & Science 1D06 Quiz #3 SOLUTIONS 23 October 2015

Full Name:________________________________ Student # :____________

TA:________________________________

Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

[6 marks] (1)
- The y-intercept is $f(0) = 1$
- $\lim_{x\to\pm\infty} f(x) = -1$
- f has a vertical asymptote at $x = -1$
- $f'(x) < 0$ for all x, and f' is never equal to zero. So no local max/min
- f' is decreasing on $(-\infty, -1)$ and increasing on $(-1, \infty)$.



[4 marks] (5) Find the critical point of the function

$$g(x) = 3^{x^2-x}.$$

Is this point a maximum, a minimum, or neither?

We find a critical point by setting the derivative equal to zero. Well, finding the derivative involves the chain rule. The inner function is $x^2 - x$, whose derivative is $(2x - 1)$. The outer function is 3^x, whose derivative is $3^x \ln(3)$. Thus

$$g'(x) = (2x - 1)3^{x^2-x} \ln(3).$$

This can only be zero when $(2x - 1) = 0$, or when $x = \frac{1}{2}$. The $y-$value of this point is $g\left(\frac{1}{2}\right) = 3^{-\frac{1}{2}} = \frac{1}{\sqrt{3}}$ To see this is a **minimum**, notice that when $x < \frac{1}{2}$, $g'(x) < 0$ (since, for example, $g'(0) = -1$) and when $x > \frac{1}{2}$, $g'(x) > 0$ (since, for example, $g'(1) = 1$). The derivative goes from negative to positive, which means the function goes from decreasing to increasing, so the point $(1, \frac{1}{\sqrt{3}})$ is a minimum.