Arts & Science 1D06 Quiz #3 22 October 2015

Full Name: SOLUTIONS Student # :

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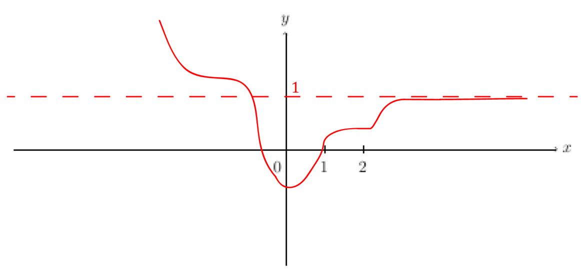
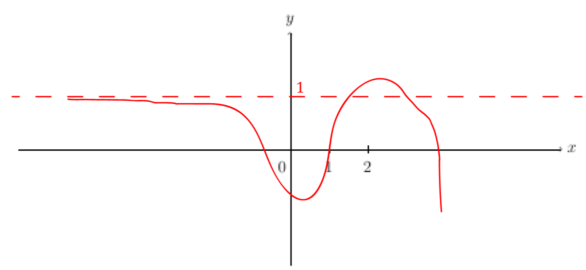
Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

[6 marks] (1) Sketch the graph of a continuous function f with the following properties:

- $f'(0) = 0$ and $f'(2) = 0$
- $f''(1) = 0$, $f''(3) = 0$, and $f''(x) \leq 0$ for $x > 1$
- f has a horizontal asymptote at $y = 1$



OR



[4 marks] (2) Find the minimum value that the function $g(x) = \frac{1}{3}x^3 - 2x^2 + 3x + 1$ assumes on the closed interval $[0, 2]$.

To find the minimum value of $g(x)$, we need to compute $g'(x)$ and then find points at which $g'(x) = 0$:

$$g'(x) = x^2 - 4x + 3 = (x-1)(x-3) = 0 \Leftrightarrow x = 1 \text{ or } x = 3.$$

Now we can disregard $x = 3$ since it is not in our interval, and evaluate our functions at the critical point $x = 1$ and at the endpoints of our interval:

$$f(0) = {}^1\!/_3(0)^3 - 2(0)^2 + 3(0) + 1 = 1$$
$$f(1) = {}^1\!/_3(1)^3 - 2(1)^2 + 3(1) + 1 = {}^7\!/_3$$
$$f(2) = {}^1\!/_3(2)^2 - 2(2)^2 + 3(2) + 1 = {}^5\!/_3$$

So the minimum of g on $[0, 2]$ is 1 at $x = 0$.