Arts & Science 1D06 Quiz #4

5 November 2015

Full Name: SOLUTIONS Student # :

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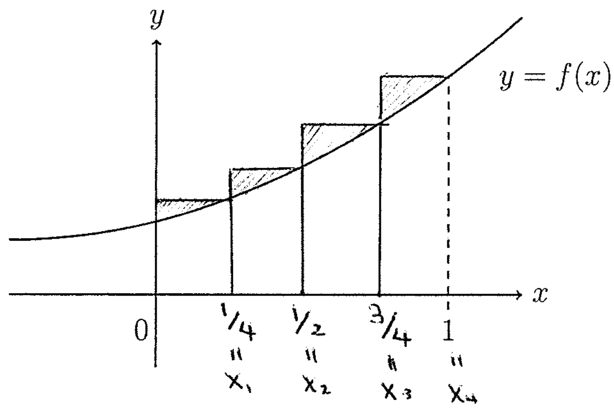
Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

[7 marks] (1) Estimate the area under the curve $f(x) = x^2 + x + 1$ on the interval $[0, 1]$ using 4 approximating rectangles and right endpoints. Is the area you calculated an overestimate or understimate for $\int_0^1 f(x)dx$? Explain your answer.



$$\boxed{\Delta x = \frac{1-0}{4} = \frac{1}{4}}$$

$$R_4 = \sum_{n=1}^{4} f(x_n)\Delta x$$

$$= \sum_{n=1}^{4} (x_n^2 + x_n + 1)\left(\frac{1}{4}\right)$$

$$= \frac{1}{4}\left(\sum_{n=1}^{4} x_n^2 + x_n + 1\right)$$

$$= \frac{1}{4}\left(\left[\left(\frac{1}{4}\right)^2 + \frac{1}{4} + 1\right] + \left[\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 1\right] + \left[\left(\frac{3}{4}\right)^2 + \frac{3}{4} + 1\right] + \left[1^2 + 1 + 1\right]\right)$$

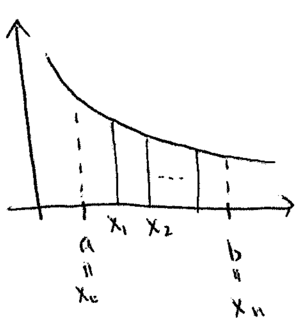
$$= \frac{1}{4}\left(\frac{21}{16} + \frac{7}{4} + \frac{37}{16} + 3\right) = \frac{67}{32}$$

This is an overestimate

↳ see diagram above

[3 marks] (2) Consider the area under the curve for a continuous function g on an interval $[a, b]$. If we call the Riemann Sum with n rectangles and left endpoints L_n and the Riemann sum with n rectangles and right endpoints R_n, is it always true that $L_n < R_n$? Explain your answer.

No. Consider a decreasing function.



Since $f(x_k) \geq f(x_{k+1})$, we will have $f(x_k)\Delta x \geq f(x_{k+1})\Delta x$

$\Rightarrow L_n \geq R_n$