Arts & Science 1D06 Quiz #5

20 November 2015

Full Name:_____ Student # :_____

TA:_____

Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

[10 marks] (1) Evaluate the following definite integrals:

(a) *[5]*

$$\int_0^1 xe^{3x^2}\,dx.$$

We have a function ($3x^2$) and its derivative (up to a constant) $x\,dx$. So we make the substitution $u = 3x^2$, and so $\frac{du}{dx} = 6x$, which means that $\frac{1}{6}du = x\,dx$. We also need to change the bounds of the integral. When $x = 0$, $u = 0$ and when $x = 1$, $u = 3$. So:

$$\begin{aligned}\int_0^1 xe^{3x^2}\,dx &= \int_0^3 \frac{1}{6}e^u\,dx \\ &= \frac{1}{6}e^u\Big|_0^3 \\ &= \frac{1}{6}\left(e^3 - 1\right)\end{aligned}$$

(b) *[5]*

$$\int_0^1 xe^{3x}\,dx.$$

This is a product of two functions (namely x and e^{3x}), so we use integration by parts. We choose one functions to integrate, and the other to differentiate. Well, x becomes simpler when differentiated, and e^{3x} stays the same either way. So we make the following table:

$$\begin{aligned} u &= x & v &= \frac{1}{3}e^{3x} \\ du &= dx & dv &= e^{3x}\,dx \end{aligned}$$

Subbing into the integration by parts formula gives:

$$\begin{aligned}\int_0^1 xe^{3x}\,dx &= \frac{1}{3}xe^{3x}\Big|_0^1 - \frac{1}{3}\int_0^1 e^{3x}\,dx \\ &= \frac{1}{3}xe^{3x}\Big|_0^1 - \frac{1}{3}\frac{1}{3}e^{3x}\Big|_0^1 \\ &= \frac{1}{3}\left(1\cdot e^3 - 0\cdot e^0\right) - \frac{1}{9}\left(e^3 - e^0\right) \\ &= \frac{2}{9}\left(e^3 + 1\right)\end{aligned}$$