Arts & Science 1D06 Quiz #5

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Full Name: SOLUTIONS Student # :

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Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

[10 marks]

(1) Evaluate the integral

$$\int_0^{\frac{\pi}{4}} \cos(2x)\sin(3x)dx$$

For our first integration by parts, we'll use

$$\begin{cases} u = \cos 2x & dv = \sin 3x dx \\ du = -2\sin 2x & v = -\frac{1}{3}\cos 3x dx \end{cases}$$

$$\begin{aligned} \int_0^{\frac{\pi}{4}} \cos(2x)\sin(3x)dx &= -\frac{1}{3}\cos 2x\cos 3x\Big|_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \left(-\frac{1}{3}\cos 3x\right)(-2\sin 2x dx) \\ &= \frac{1}{3}(0-1) - \frac{2}{3}\int_0^{\frac{\pi}{4}} \cos 3x\sin 2x dx \\ &= \frac{1}{3} - \frac{2}{3}\int_0^{\frac{\pi}{4}} \cos 3x\sin 2x dx \end{aligned}$$

Now we'll use integration by parts again

$$\begin{cases} u = \sin 2x & dv = \cos 3x dx \\ du = 2\cos 2x & v = \frac{1}{3}\sin 3x dx \end{cases}$$

$$\begin{aligned} \int_0^{\frac{\pi}{4}} \cos(2x)\sin(3x)dx &= \frac{1}{3} - \frac{2}{3}\int_0^{\frac{\pi}{4}} \cos 3x\sin 2x dx \\ \int_0^{\frac{\pi}{4}} \cos(2x)\sin(3x)dx &= \frac{1}{3} - \frac{2}{3}\left[\frac{1}{3}\sin 2x\sin 3x\Big|_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \left(\frac{1}{3}\sin 3x\right)(2\cos 2x dx)\right] \\ \int_0^{\frac{\pi}{4}} \cos(2x)\sin(3x)dx &= \frac{1}{3} - \frac{2}{9}\left(\frac{\sqrt{2}}{2} - 0\right) + \frac{4}{9}\int_0^{\frac{\pi}{4}} \cos(2x)\sin(3x)dx \\ \frac{5}{9}\int_0^{\frac{\pi}{4}} \cos(2x)\sin(3x)dx &= \frac{1}{3} - \frac{\sqrt{2}}{9} \\ \int_0^{\frac{\pi}{4}} \cos(2x)\sin(3x)dx &= \frac{9}{5}\left(\frac{1}{3} - \frac{\sqrt{2}}{9}\right) = \frac{1}{5}\left(3 - \sqrt{2}\right) \end{aligned}$$