Arts & Science 1D06 Quiz #6 4 December 2015

Full Name:______________________________ Student # :________________

TA:______________________________

Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

(a) *[5]*

$$\int \frac{1}{x(x^2+1)}\,dx.$$

Once again using partial fraction decomposition:

$$\frac{1}{x(x^2+2)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}.$$

(I use this form for partial fraction decomposition since the second term in the denominator is a quadratic that can't be factored.) Multiplying both sides by $x(x^2+2)$ gives

$$1 = A(x^2+1) + (Bx+C)x.$$

Subbing in $x=0$ means that $1 = A(0+1) + (B(0)+C)(0) \implies A = 1$. This means that $1 = x^2+1+Bx^2+Cx$. Plugging in $x=1$ and $x=-1$ in turn give:

$$\begin{aligned} 1 &= 1+1+B+C \implies B+C=-1 \\ 1 &= 1+1+B-C \implies B-C=-1 \end{aligned}$$

These two equations can only both be true if $C=0$ and $B=-1$. The partial fraction decomposition is therefore

$$\frac{1}{x(x^2+1)} = \frac{1}{x} - \frac{x}{(x^2+1)}.$$

Plugging this back into the integral:

$$\begin{aligned} \int \frac{1}{x(x^2+1)}\,dx &= \int \frac{1}{x}\,dx - \int \frac{x}{x^2+1}\,dx \\ &= \ln|x| - \frac{1}{2}\ln(x^2+1) + C \end{aligned}$$

(b) *[5]*

$$\int \frac{2x-1}{(x+4)^2}\,dx.$$

I'll use partial fraction decomposition to write

$$\frac{2x-1}{(x+4)^2} = \frac{A}{x+4} + \frac{B}{(x+4)^2}.$$

(I use this form for partial fraction decomposition since I've got a repeated linear factor.) Multiplying both sides by $(x+4)^2$ gives:

$$2x - 1 = A(x+4) + B.$$

I can now sub in $x = -4$, which gives $2(-4) - 1 = A(0) + B \implies B = -9$. Next I'll plug in $x = 0$, which means that $2(0) - 1 = 4A - 9 \implies A = 2$. Putting this decomposition back into the original integral:

$$\begin{aligned}\int \frac{2x-1}{(x+4)^2}\,dx &= \int \frac{2}{(x+4)}\,dx - \int \frac{9}{(x+4)^2}\,dx \\ &= 2\ln|x+4| + \frac{9}{(x+4)} + C\end{aligned}$$