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Please provide detailed solutions to the problems below. Correct responses without justification may not receive full credit. The use of a calculator is permitted.

[10 marks] (1) Using integration by partial fractions, find the most general antiderivative of

$$f(x) = \frac{x^2 + x + 4}{(x+2)(x^2+2)}$$

$$\frac{x^2+x+4}{(x+2)(x^2+2)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+2}$$

$$A(x^2+2) + (Bx+C)(x+2) = x^2+x+4$$

$$(A+B)x^2 + (2B+C)x + (2A+2C) = x^2+x+4$$

$$\begin{cases} A+B=1 & \textcircled{1} \\ 2B+C=1 & \textcircled{2} \\ 2A+2C=4 & \textcircled{3} \end{cases} \Rightarrow \boxed{A=1, B=0, C=1}$$

$$\textcircled{1} \quad B=1-A \quad \& \quad \textcircled{3} \quad C=2-A$$

$$\text{so } \textcircled{2} \quad 2(1-A) + 2 - A = 1 \Leftrightarrow 2 - 2A + 2 - A = 1$$

$$3A = 3 \Leftrightarrow A = 1$$

$$\begin{aligned} \text{so } \int \frac{x^2+x+4}{(x+2)(x^2+2)} dx &= \int \left( \frac{1}{x+2} + \frac{1}{x^2+2} \right) dx \\ &= \int \frac{dx}{x+2} + \int \frac{\frac{1}{2}}{1 + \left(\frac{x}{\sqrt{2}}\right)^2} dx \\ &= \int \frac{dx}{x+2} + \frac{1}{2} \int \frac{dx}{1 + \left(\frac{x}{\sqrt{2}}\right)^2} \end{aligned}$$

$$= \ln|x+2| + \frac{1}{2} \cdot \sqrt{2} \arctan\left(\frac{x}{\sqrt{2}}\right) + C$$

$$= \ln|x+2| + \frac{1}{\sqrt{2}} \arctan\left(\frac{x}{\sqrt{2}}\right) + C$$