ArtSci 1D06 Calculus full year 2015–2016

Instructor: D. Haskell

Practice problems towards Midyear Exam

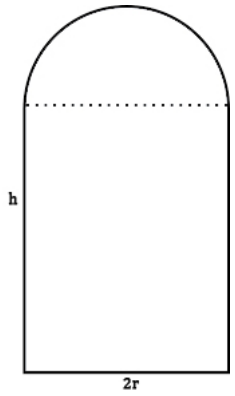
The following sixteen problems are representative of the kind of problems that you might get on the exam. These problems will certainly take you more than two and a half hours (the alotted time) to complete. The formula sheet that you will get in the exam is also included.

1) Using Newton's Method, approximate $\sqrt[5]{28}$ to 3 decimal places with initial approximation is $x_1 = 2$.

2) Evaluate the following limits. Use L'Hôpital's Rule if applicable.
   a) $\lim\limits_{x\to 0} \dfrac{e^x}{1-\cos x}$
   b) $\lim\limits_{x\to 0} \dfrac{\cos(x)-1}{e^x-1}$
   c) $\lim\limits_{x\to \infty} x^{1/x}$
   d) $\lim\limits_{x\to \infty} \dfrac{3x^2+7x-16}{x^2-4}$

3) Determine the area bounded by $y = x - 1$ and $y^2 = 2x + 6$.

4) Dr Haskell is renovating her house and wants to build a window where the bottom is a rectangle and the top is a semicircle, as shown below. If Dr Haskell only bought 12 metres of framing material from Canadian Tire, what dimensions should the window have to let in the most light?



5) Do all the calculations needed to fill in the following table and then sketch the graph of $f(x) = e^{-x^2}$.

Domain of f:	
x-intercept:	
y-intercept:	
Symmetries:	
Horizontal Asymptotes:	
Critical points:	
f is increasing on:	
f is decreasing on:	
Inflection points:	
f is concave up on:	
f is concave down on:	

6) (a) Compute $\lim_{x\to 0} x^x$ (Hint: use a logarithm, then l'Hôpital's Rule).
   (b) What are the minimum and maximum values of $y = x^x$ on the interval $(0, 1/2]$?

7) Consider the function $h(x) = \sqrt{1-x^2} - \sinh(x)$.
   (a) State the Intermediate Value Theorem.
   (b) Does $h(x)$ have a solution on the interval $(0, 1)$?
   (c) Compute $h'(x)$.
   (d) Compute $\int h(x)\,dx$.
   (e) Let $H(x)$ be the anti-derivative of $h(x)$ that satisfies $H(0) = 1$. Write the formula for $H(x)$.

8) True or false (justify your answers):
   a) If a function is integrable on the interval (a, b) then it's continuous on the interval (a, b).
   b) If $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$ then the function $f(x)$ is continuous at the point a.

9) Suppose that the function f is differentiable on $[0, 1]$. Is it true that there is a number c in $[0, 1]$ such that $f'(c) = f(1)$?

10) Let $F(x) = \int_{\sin(x)}^{\cos(x)} e^{-x^2}\,dx$. Compute $F'(x)$.

11) Show that the equation $x + e^x = 0$ has exactly one solution.

12) For what values of c is the function $g(x) = cx + \dfrac{1}{x^2+3}$ increasing on $(-\infty, \infty)$?

13) Find the following integrals.
   a) $\displaystyle\int \frac{x+7}{x^2(x+2)}\,dx.$
   b) $\displaystyle\int \frac{x^2}{\sqrt{1+x^2}}\,dx.$
   c) $\displaystyle\int \sin(2x)e^{5x}\,dx.$
   d) $\displaystyle\int \frac{\sin(\ln(x))}{x^2}\,dx$
   e) $\displaystyle\int_0^\infty \frac{e^{1/x}}{x^2}\,dx.$

14) Find the length of the curve $y = \cosh(x)$ from $x = 0$ to $x = 1$.

15) Find the volume of the solid formed by rotating the graph of $y = \sin(x)$ around the x-axis from $x = 0$ to $x = \pi$.

16) Calculate an upper bound and a lower bound to the area under the graph of $y = \dfrac{1}{x}$ from $x = 1$ to $x = 2$ using left and right Riemann sums with four subintervals. Draw a picture for each to decide which is the upper bound and which is the lower bound. Then calculate the integral explicitly.

Formula Sheet

Integrals (constants of integration are omitted)

$$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1 \qquad \int \frac{1}{x}\,dx = \ln|x|$$

$$\int e^x\,dx = e^x \qquad \int a^x\,dx = \frac{a^x}{\ln a}$$

$$\int \sin x\,dx = -\cos x \qquad \int \cos x\,dx = \sin x$$

$$\int \tan x\,dx = -\ln|\cos x| \qquad \int \cot x\,dx = \ln|\sin x|$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| \qquad \int \csc x\,dx = -\ln|\csc x + \cot x|$$

$$\int \sec^2 x\,dx = \tan x \qquad \int \csc^2 x\,dx = -\cot x$$

$$\int \sec x\tan x\,dx = \sec x \qquad \int \csc x\cot x\,dx = -\csc x$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) \qquad \int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right)$$

Trigonometry

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

$$\sin(2x) = 2\sin x\cos x \quad \cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1$$

Newton's method $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Arc length $L = \displaystyle\int_a^b \sqrt{1+[f'(x)]^2}\ dx$