MATH 790 — MAJOR RESEARCH PROJECT
Project Topics for 2016–2017 (preliminary version)

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Stanley Alama

My research involves the Calculus of Variations and Nonlinear Functional Analysis in the study of nonlinear Partial Differential Equations. These nonlinear problems typically arise in the modeling of natural phenomena (in physics, biology, or even economics,) or in differential geometry. My main interest lies in the development and application of methods of mathematical analysis to these scientific contexts. For master's students seeking a project for Math 790, there are many directions to pursue. From the point of view of PDE, we are interested in the existence, uniqueness, and qualitative properties of solutions. In the Calculus of Variations, the aim is to develop tools for finding critical points of functions on infinite dimensional spaces. In the direction of mathematical physics, there are questions concerning quantized vortices in superconductivity or liquid crystals, and problems of phase separation in ferromagnetism. A typical problem will combine elements from each area, using techniques of Nonlinear Functional Analysis. An appropriate preparation would include Lebesgue integration (Real Analysis 721), some Complex Analysis (722 or 4X3), and some PDE (742). The ideal would be Functional Analysis (723, offered this winter.)

Sample problems: Symmetry of least-energy solutions to a nonlinear PDE; Uniqueness of solutions to systems of nonlinear PDE; Symmetry-breaking bifurcation of solutions to a boundary-value problem. Any of the above, applied to vortices, domain walls, monopoles, etc. I would also be interested in exploring some of the history, or alternative formulations, of some classical results in analysis, such as P. Lax's recent A.M.M. article on deriving Lebesgue integration without constructing the measure.

Hans Boden

My research interests lie in the general area of geometric topology, and I work on problems in gauge theory and low-dimensional topology. This include the study of knots, links, and 3-dimensional manifolds, and as well as using moduli spaces and/or character varieties to define invariants. Virtual knot theory is an area where there are lots of interesting open problems that are suitable for an MSc project or thesis. Some suggested MSc project topics: 1. The A-polynomial and $SL(2,\mathbb{C})$ representations of knot groups; 2. arithmetic in the braid group, 3. Khovanov homology and knot Floer homology, 4. classical and virtual knot concordance. Other ideas are welcome, and interested students are encouraged to contact me in person (HH/218E) or by email (`boden@mcmaster.ca`).
Possible number of students: 1 MSc (Project or Thesis).

Ben Bolker

I am interested in supervising projects relating to biological population dynamics (ecological,

evolutionary, and epidemiological) and biostatistics. Examples include various problems in ecological estimation; theoretical models of the evolutionary dynamics of virulence (mostly based on ordinary differential equations); and implementation and testing of general tools for generalized linear mixed models (GLMMs). It would be useful but not essential to have taken some introductory biology (especially ecology or evolution). Preferred math background includes at least 1B, 2X, 2C. Some programming experience is also strong preferable, ideally in R but familiarity with MATLAB, Java, FORTRAN, or C++ would be reasonable substitutes.
Possible number of students: 1 or 2

Lia Bronsard

In the field of Partial Differential Equations and the Calculus of Variations, we are interested in studying qualitative properties of solution of equations arising in various contexts: physics, geometry, biology, or economics, for instance. For example, the Allen-Cahn equation models the evolution of different phases in a material. In a certain limit, the boundary curve (or surface) separating two phases evolves according to its curvature. In this way, analysis, physics, and geometry enter into the study of this equation. Another example is the study of vortices (point singularities) of solutions to the Ginzburg-Landau model of superconductors. This is a problem in the Calculus of Variations, in which one seeks critical points on infinite dimensional spaces of functions. Using methods of real analysis (Math 4A03-6A03, Math 721) we may understand the qualitative properties of solutions, as well as studying the existence, uniqueness and regularity of solutions. Some possible areas of study are: traveling and stationary wave solutions to reaction-diffusion equations; Gamma-convergence and geometrical optimization problems; vortex profiles and their stability. I would also be happy to discuss other directions of research in differential equations or analysis to match the interests of the student.

Jonathan Dushoff

I am a quantitative biologist with broad interests in the evolution and spread of infectious diseases, and other ecological and evolutionary questions. To work with me you should be interested in *scientific* applications of mathematics, and ready to talk about which scientific questions interest you. You do not need formal biological training. Possible applications include: modeling disease spread in a population; modeling the interaction between cancer cells and immune cells in an individual; modeling the evolution of a virus in one or more populations.

David Earn

The sorts of projects that I am normally interested in supervising are connected with epidemiology of infectious diseases, conservation of endangered species, or the evolution of animal behaviour. One of the my primary research projects at the moment concerns disease dynamics in London, England, over the last four hundred years. However, if you are interested in another area of biology, and have an idea that you would like to investigate using mathematical models, then I would be very happy to discuss it. You might think that to work with me you need a formal background in biology, but that's not true. You just need to be interested in biology. You do need to have taken some mathematics courses and, most importantly, you need to be keen to use mathematical models to contribute to our understanding of biology. Some prior knowledge of computer programming is usually essential; in any case, willingness to learn to use R and/or XPPAUT is critical.

Typical Math 790 projects would involve analysis of data (e.g., see

`http://iidda.mcmaster.ca`

for epidemiological data digitized by my research group) and mathematical modelling aiming to understand the temporal and/or spatial patterns in these data.

Minimum course requirements: Math 3F03 or equivalent, i.e., an undergraduate course covering the qualitative theory of differential equations.

Recommended courses: Math 741 (Applied Math), 746 (Bifurcation Theory), 747 (Mathematical Biology).

Possible number of students: 1

Jean-Pierre Gabardo

Time-Frequency analysis: Basic Fourier analysis, Short-time Fourier transform, Quadratic Time-Frequency representations, Gabor frames.

Ian Hambleton

I would like to meet and discuss possible MSc projects or thesis topics in geometry, topology or algebra. Some of my areas of interest are algebraic number theory, group actions, geometric group theory, and the topology of manifolds. Recommended courses: any of the 600 or 700 level courses offered in this area. For 2016-2017 the options include: Math 701, Math 702, Math 761, Math 762, Math 766.

Possible number of students: 1 MSc (Project or Thesis).

Megumi Harada

My research is in equivariant symplectic and algebraic geometry in a broad sense, as well as their connections to other areas such as geometric representation theory and equivariant algebraic topology. I am particularly interested in using combinatorical techniques to answer geometric and topological questions. Particular examples of topics of recent interest to me are toric varieties and Okounkov bodies, Schubert calculus, and Hessenberg varieties. Many of these topics have introductory treatments (lecture notes, textbooks, ex- pository articles) which would be accessible to an advanced and motivated undergraduate or beginning graduate student; they also have a wealth of open problems which would be amenable to explicit computational exploration, suitable for a summer or Math 790 project. However, the actual topic would be tailored to student's interests and background.

Computer programming skills, using e.g. the mathematical software SAGE, is highly recommended.

Minimum course requirements: Math 3E. Recommended: Math 3EE, algebraic topology, basic differential geometry, some familiarity with algebraic geometry.

Possible number of students: 1

Bradd Hart

My research area is mathematical logic and its applications. There could be several interesting projects/essays based on problems of a logical nature like the status of the continuum problem, Newton's infinitesimals or decidability and undecidability results and their impact on other areas of mathematics. A basic undergraduate course in logic would be a good starting point. Feel free to stop by if you are interested in doing a project with me.

Nicholas Kevlahan

Adaptive simulation of atmosphere and ocean flows In this project the student will review the current non-adaptive numerical methods used to simulate atmosphere and ocean flows (e.g. for weather and climate modelling), and investigate the methods currently proposed to make these models adaptive (e.g. adaptive mesh refinement (AMR), adaptive wavelet methods, discontinuous Galerkin). The student will identify the main challenges for these models, and summarize their strong and weak points. Ideally, the student will use a standard non-adaptive numerical code for the shallow water equations (based on spherical harmonics) to investigate the properties of a range of benchmark flows.

The student should have some experience with programming and have taken at least one course in numerical analysis, and ideally have some understanding of geophysical fluid dynamics.

Miroslav Lovric

Mathematics Models in Biology: Research and Creation of Teaching Materials

Investigation of models in medicine and biology (such as biological applications of diffusion, nerve impulses, cancer growth) and development of teaching materials based on the research.

1 student.

Andrew Nicas

My research specialties are algebraic topology, geometric topology and geometric group theory. M.Sc. projects under my supervision could cover a wide variety of topics, for example: knot and link theory, the large scale geometry of metric spaces with applications to the study of infinite groups, and real life uses of algebraic topology (shape recognition, sensor networks). I encourage students to stop by my office, HH-310, to discuss possible projects. Some background in undergraduate algebra, similar to our MATH 3E03, and some basic real analysis and topology would be useful prerequisites.

Dmitry Pelinovsky

My research interests are in nonlinear partial differential equations and discrete dynamical systems considered in the context of applied physical problems. In particular, I am currently interested in dynamics of granular crystals, justification of nonlinear Schrodinger equations for polymers, bifurcations of rotating vortices in Bose-Einstein condensates with harmonic potentials, dynamics of shock waves in viscous Burgers equations, and the inverse scattering transform in the context of asymptotic stability of solitary waves. A project with the focus on the literature review and computational aspects for any of the problem above is possible for one or two M. Sc. students with the background in applied mathematics, differential equations, and numerical analysis.

Bartosz Protas

Topics in Vortex Dynamics

One of the main research themes in my group concerns problems of vortex dynamics. Vortices are a class of solutions of equations describing the motion of an inviscid fluid. As such they represent idealized models of many natural phenomena ranging from swirling motion in the bathtub to cyclones occurring on the continental scale. The principal open questions in this field are related to the existence of equilibrium solutions in domains with different geometries, stability of such equilibria and also methods for controlling vortex motion. These problems are typically addressed using a combination of mathematical analysis and methods of scientific computing.

The goal of a project undertaken as a part of the course MATH 790 would be to explore some classical and some open problems in vortex dynamics. As a first step, the student would need to do some reading to acquire necessary background in theoretical fluid mechanics and computational mathematics. The main part of the project will involve development of computational techniques using MATLAB and/or Maple. These tools will then be applied to investigate the properties of some selected vortex systems combining analytical and computational insights.

Minimum course requirements: Math 3F03, Math 3Q03 (any of Math 2T03, Math 4G03 and Math 4Q03 would also be useful.)
Possible number of students: 1

Matthew Valeriote

Algebra, Logic and the Constraint Satisfaction Problem

Many interesting and important questions from computer science, combinatorics, logic, and database theory can be expressed in the form of a constraint satisfaction problem. Recently an algebraic approach to settling some central conjectures in this area have been developed and have led to the investigation of some novel properties of finite algebraic systems.

The proposed project will involve experimenting with small algebraic systems to test several conjectures and open problems that are concerned with the existence of certain derived operations of an algebra that help to govern the types of solutions that may exist for associated instances of the constraint satisfaction problem. There are a couple of computational tools that may be employed in this project, but during the initial phase of the project a fair amount of background reading will need to be done.

Adam Van Tuyl

My principal area of research is commutative algebra. I am especially interested in the interplay between commutative algebra and algebraic geometry, and more recently, the interplay of this field with combinatorics, e.g, graph theory. My work is primarily algebraic in that I am interested in the structure and numerical invariants of rings that arise from algebraic geometry and combinatorics. Specific topics that I am currently investigating include monomial ideals, simplicial complexes, symbolic powers of ideals, and combinatorial matrix theory. Many of the problems in which I am interested have a facet that can be explored computationally, and would provide a basis for a Math 790 project. Any project would be developed in consultation with the student, taking into account both the interests and the background of the student.

An undergraduate course in abstract algebra would be a minimum course prerequisite, and it would be expected that students enroll in Math 701/702, the core graduate algebra sequence. Interested students are more than welcome to drop by my office or send me an email.

Roman Viveros-Aguilera

Industrial statistics refers to a collection of statistical methods applicable in the solution of problems in industrial settings. The problems may involve assessing the reliability of a product,

modeling failure mechanisms, assessing the effect of different factors on the reliability, or monitoring and controlling quality in a manufacturing or service process. I will be delighted to work with a student who has an interest in one aspect of industrial statistics. It could be studying a particular application or learning about some of the methodologies. The specific program to follow will depend on the students interest. Minimum course requirements: STATS 3A3
Possible number of students: 1

McKenzie Wang
Simply put, Differential Geometry is the the study of geometry using the techniques of calculus and linear algebra and their more sophisticated generalizations such as the theory of ordinary and partial differential equations, Lie group theory, and differential topology. Students interested in projects in Differential Geometry should preferably have some prior exposure to geometry at the undergraduate level, for example through a course on curves and surfaces in $\mathbb{R}^3$, or a course on calculus on manifolds. Absent this, some knowledge of complex analysis, dynamical systems, topology, and/or mathematical physics together with a willingness to learn geometry and topology "on the job" could be sufficient preparation. I welcome interested students to discuss their background with me and I will try to find interesting projects for them. For those with an intention to specialise in Differential Geometry later, taking the sequence Math 761/762 concurrently is strongly recommended.

The following is just a small sample list of many possible topics. A project can also involve learning about some area of geometry, e.g. surfaces of constant mean curvature, mean curvature flow, through reading some papers from the literature. A list of my former students and their projects/theses can be found in my personal webpage.

1. **Explore the dynamics of selected symmetry reductions of the Einstein equations to ODEs.** This project involves using a combination of dynamical systems methods and numerical methods to study the dynamics of the Einstein equation $\mathrm{Ric}(g) = \lambda g$ when g has a large symmetry group. Besides the Einstein equation, one can also look at generalizations of the Einstein equations such as the quasi-Einstein equations and the Ricci soliton equations. The possibilities are numerous and often the results are of interest in mathematical physics.

2. **Explore integrability issues for symmetry reductions of the Einstein equation.** The simplest meaning of integrability is having solutions given by explicit formulae. However, integrability is also related to the existence of sufficiently many Poisson commuting first integrals (conserved quantities), the existence of sufficiently many Painleve expansions with movable singularities, the existence of Lax pairs, or to the existence of superpotentials. These phenomena lend themselves to study by symplectic and/or algebraic geometry methods, and also have relations to Hamilton-Jacobi equations.

3. **A problem involving finite subgroups of compact Lie groups.** The general question here is to find examples of finite subgroups $\Gamma \subset G$ of a compact simple Lie group G with the property that the adjoint representation of G remains irreducible when restricted to Γ. One would like to classify these subgroups if possible. Known examples include the icosahedral group in SO(3) and the Harada-Norton group in the orthogonal group O(133).

Gail S. K. Wolkowicz

My research focuses on the development and analysis of mathematical models to study population dynamics. One goal is to reconcile what are commonly believed general principles with conflicting experimental observations, and hence suggest new or modified principles. Another goal is to develop measurable criteria that would enable scientists to predict which combination of microorganisms would be most effective and safest for use in such processes as water purification, biological waste decomposition, and biological remediation. Other potential applications include pest control on the one hand and the prevention of the extinction of endangered species on the other. Recently I have become interested in modelling: anaerobic digestion related to the production of green energy from animal waste; the prevalence of antibiotic resistant strains of bacteria and how to develop measures to control their spread; different aspects of spread and control of Dengue fever, a mosquito borne disease responsible for infecting 50-100 million people annually resulting in 10,000 infant deaths.

Formal training in biology is not necessary. Mathematics courses such as M3MB3, M3F03, M4MB3, and 4A03 at the undergrad level would be helpful, but are not necessary. Students working with me at the graduate level are expected to be enrolled in M6A03 and M741 or to have already taken the equivalent. My students will learn to use the qualitative theory of differential equations to determine the local and when possible global dynamics of the models under consideration. They will learn to use persistence theory when appropriate. Bifurcation theory will help to determine the full spectrum of behaviour for all appropriate parameter ranges and initial states and help to identify which key parameters need to be measured and how accurately, since only changes in their magnitude result in changes in the dynamics. If there are significant time delays involved in any of the interactions, integro and functional differential equations theory will be used. Students will develop their computing skills. Computer simulations will be used to elucidate complicated dynamics, to test conjectures and to reveal properties of the models that are useful in developing analytic proofs. Symbolic computation will be used for complicated calculations. Specialized software for obtaining bifurcation diagrams will be especially useful to help understand how changes in parameters can affect model dynamics. The analysis will often lead to interesting abstract mathematical problems in dynamical systems, ordinary, impulsive, integro, and functional differential equations, and the qualitative theory of differential equations.