STAT743 FOUNDATIONS OF STATISTICS (Part I)

Assignment 2 Due in class at **12:30pm on Thursday October 28, 2019**

Instructions:

1. Ensure that your full name and student number are clearly marked on each page of your solutions.

2. Solutions need not be typed (although they can be) but must be readable.

3. Start each question on a new page and clearly indicate where each part of the question begins.

4. Submit questions in the same order as given below.

5. You are expected to show all details of your solution and any results taken from my notes or the textbook must be clearly and properly referenced.

6. No extensions to the due date and time will be given except in extreme circumstances and late assignments will not be accepted. Assignments are due at the **start** of class.

7. Students are reminded that submitted assignments must be their own work. Submission of someone else's solution under your name is academic misconduct and will be dealt with as such. Penalties for academic misconduct can include a 0 for the assignment, an F for the course with an annotation on your transcript and/or dismissal from your program of study.

Q. 1 **a)** Suppose that X is a standard normal random variable. Find the pdf of the random variable $Y = |X|$ and find the mean and variance of Y.

b) Casella and Berger 4.19

Q. 2 **a)** Casella and Berger 4.26

b) Casella and Berger 4.30

Q. 3 Suppose that X is a standard normal random variable and U is a uniform$(0, 1)$ random variable independent of X. Define the random variable

$$Y = \begin{cases} X & \text{if } U < 0.5 \\ -X & \text{if } U \geqslant 0.5 \end{cases}$$

a) Show that Y is also a standard normal random variable.

b) Show that the correlation coefficient of X and Y is equal to 0 but that they are not independent.

c) Find the conditional distribution of Y given $X = x$ and hence show that the random vector (X, Y) cannot be bivariate normal even though both marginal distributions are normal.

Q. 4 **a)** Suppose that X and Y are independent Geometric(θ) random variables and we define

$$U = \min\{X, Y\} \qquad V = X - Y$$

Show that U and V are independent and give their marginal probability mass functions.

b) Consider the hierarchical model given by

$$Y \mid \Lambda = \lambda \sim \text{Poisson}(\lambda) \qquad \Lambda \sim \text{gamma}(\alpha, \beta)$$

(i) Find the marginal probability mass function of Y and show that when α is an integer the marginal distribution of Y is negative binomial.

(ii) Find the mean and variance of Y and show that we can write

$$\text{Var}(Y) = \text{E}(Y) + \frac{1}{\alpha}\big(\text{E}(Y)\big)^2$$

and compare these results to the case of a fixed parameter λ for the Poisson random variable.