Disclaimer: *The following solutions are most likely correct. In the event of a discrepency, please inform the professor as soon as possible!*

Solutions: Test #2b

Multiple choice answers: *(One mark each)*

1. a)   2. b)   3. d)   4. a)   5. e)   6. c)   7. d)   8. d)

Long Answers: *(Three marks each)*

9. {*3 marks*}

$$y = \frac{x^2}{\sqrt{x^2 - x}} = \frac{f(x)}{g(x)},$$ so we use quotient rule:

$$y' = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2},$$ which could get messy. {1 *mark*}

But we only need $y'\big|_{x=2}$ so its faster to compute each of $f(2), g(2), f'(2), g'(2),$ and plug them in.

$$\left.\begin{array}{ll} f(2) = 2^2 = 4 & g(2) = \sqrt{2^2 - 2} = \sqrt{2} \\ f'(x) = 2x \Rightarrow & f'(2) = 2 \cdot 2 = 4 \end{array}\right\} \quad \{\tfrac{1}{2}\ \textit{mark}\}$$

$$g'(x) = \frac{d}{dx}\sqrt{x^2 - x} = \frac{d}{dx}\left(x^2 - x\right)^{1/2} = \frac{1}{2}\left(x^2 - x\right)^{-1/2}\frac{d}{dx}\left(x^2 - x\right)$$ [*By generalized power rule*]

$$= \frac{1}{2}\left(x^2 - x\right)^{-1/2}(2x - 1) \qquad \{1\ \textit{mark}\}$$

$$\Rightarrow \quad g'(2) = \frac{1}{2}\left(2^2 - 2\right)^{-1/2}(2 \cdot 2 - 1) = \frac{1}{2} \cdot \frac{1}{\sqrt{2}} \cdot 3 = \frac{3\sqrt{2}}{4}$$

Putting all this back into y' we get:

$$y'\big|_{x=2} = \frac{f'(2)g(2) - g'(2)f(2)}{(g(2))^2} = \frac{4 \cdot \sqrt{2} - \frac{3\sqrt{2}}{4} \cdot 4}{\left(\sqrt{2}\right)^2} = \frac{\sqrt{2}}{2} \doteq 0.7071 \quad \{\tfrac{1}{2}\ \textit{mark}\}$$

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10. {2+0.5 *marks*}

Note: *Due to possible clarity issues, with the wording, the last question is now out of 2, (with a possible 0.5 bonus), not the original 3.*

$$h'(x) = \frac{d}{dx}\left(x + \frac{4}{x}\right) = \frac{d}{dx}x + 4\frac{d}{dx}\left(\frac{1}{x}\right) = 1 - \frac{4}{x^2} = \frac{x^2 - 4}{x^2} \quad \{1\ mark\}$$

If $h'(x) = 0 \Rightarrow \ x = \pm 2 \}$ These are c.p.'s

If $h'(x) = DNE \Rightarrow \ x = 0 \}$ Not a c.p. since 0 is NOT in the domain of $h(x)$. $\{1\ mark\}$

Now we can check our intervals for the signs of the derivative (First derivative test):

$$\left.\begin{array}{l} \text{On the interval } x \in (-\infty, -2), h'(x) > 0 \\ \text{On the interval } x \in (-2, 0), \quad h'(x) < 0 \end{array}\right\} \Rightarrow \text{ local max at } x = -2$$

$$\left.\begin{array}{l} \text{On the interval } x \in (0,\ 2), \quad h'(x) < 0 \\ \text{On the interval } x \in (2, \infty), \quad h'(x) > 0 \end{array}\right\} \Rightarrow \text{ local min at } x = +2 \qquad \{\tfrac{1}{2}\ mark\}$$

Or, equivalently, we can perform the second derivative test:

$y'' = \dfrac{8}{x^3}$, and we evaluate at each c.p.

$y''\big|_{x=-2} = -1 < 0 \Rightarrow$ Concave Down $\Rightarrow$ local max at $x = -2$

$y''\big|_{x=-2} = +1 > 0 \Rightarrow$ Concave Up $\Rightarrow$ local min at $x = +2$

Total grades: 13