Disclaimer: *The following solutions are most likely correct. In the event of a discrepency, please inform the professor as soon as possible!*

Solutions: Test #2c - Practice

Multiple choice answers: *(One mark each)*

1. b) 2. d) 3. b) 4. a) 5. d) 6. d) 7. X 8. e)

7. Notice that #7 has no correct solution! (Oops!)
Here's how #7 goes:

We are given the equation that: $C = 12p^3 - 6p + 20$
Here C is the cost/widget and p is the cost per g of rubber.
To find the rate of increase of C over time, we differentiate everything by t.

$$C' = 36p^2 p' - 6p' + 0 = 36p^2 p' - 6p'$$

(*Remember, p is a function of time here as well, so we're using generalized power rule. Think related rates!*)
We're given that currently $p = 5\$/\text{g}$, and $p' = 0.5\$/\text{g}\cdot\text{yr.}$, so we plug these in:

$$C' = 36\cdot 5^2\cdot 0.5 - 6\cdot 0.5 = 447.00\ \$/\text{yr} \quad (\textit{Yikes}!)$$

Long Answers: *(Three marks each)*

9. {*3 marks*}

$$h(x) = \frac{x+2}{x^2-4} \Rightarrow \quad x = -2, 2 \text{ make the denominator zero} \Rightarrow \text{ possible vertical asymptotes}$$
{1 *mark*}

$$\lim_{x\to -2}\frac{x+2}{x^2-4} = \lim_{x\to -2}\frac{x+2}{(x-2)(x+2)} = \lim_{x\to -2}\frac{1}{(x-2)} = -\frac{1}{4} \neq \pm\infty \Rightarrow \text{ not a VA} \quad \{1\ \textit{mark}\}$$

$$\lim_{x\to 2}\frac{x+2}{x^2-4} = \lim_{x\to 2}\frac{x+2}{(x-2)(x+2)} = \lim_{x\to 2}\frac{1}{(x-2)} = \frac{1}{0} = ?? \Rightarrow \text{ Check left and right limits}$$

$$\Rightarrow \begin{cases} \displaystyle\lim_{x\to 2^-}\frac{x+2}{x^2-4} = \lim_{x\to 2^-}\frac{1}{(x-2)} = \frac{1}{0^-} = -\infty \\ \displaystyle\lim_{x\to 2^+}\frac{x+2}{x^2-4} = \lim_{x\to 2^+}\frac{1}{(x-2)} = \frac{1}{0^+} = +\infty \end{cases} \quad \text{So we have a VA at } x = 2.\ \{1\ \textit{mark}\}$$

Disclaimer: *The following solutions are most likely correct. In the event of a discrepency, please inform the professor as soon as possible!*

10. {*3 marks*}

$h(x) = 3x^5 - 5x^4 + 7x + 2 \Rightarrow \; h'(x) = 15x^4 - 20x^3 + 7 \Rightarrow \; h''(x) = 60x^3 - 60x^2$ {1 *mark*}

And we can factor, resulting in: $h''(x) = 60x^2(x - 1)$
Clearly then, $x = 0$ and $x = 1$ are possible inflection points, since $h''(x) = 0$ at these points. (Note: $h''(x)$ is defined everywhere, so "DNE" isn't relevant.) {1 *mark*}

We can check concavity by looking at possible $h''(x)$ values on each interval, but notice, $60x^2$ is always positive when away from $x = 0$. So by inspection we see $h''(x) > 0$ for $x > 1$ and $h''(x) < 0$ for $x < 1$ (*except when it is zero at* $x = 0$). {1 *mark*}