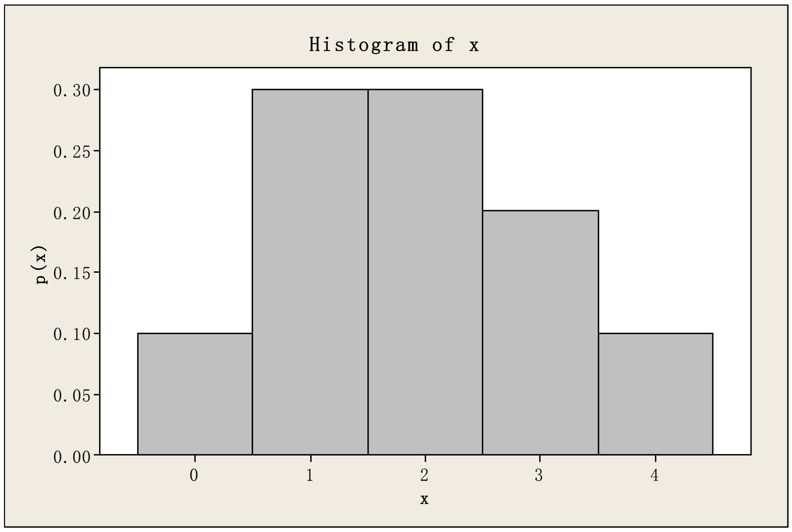
Arts/Sci 2R06 Solution for Assignment 5

4.83

a) Since one of the requirements of a probability distribution is that sum of probability of x is equal to 1, so that p(3)= 1-(0.1+0.3+0.3+0.1) = 0.2

b) Probability histogram:



c) For the random variable x given here, $E(X) = \sum X\, P(X)$

$$=0(0.1)+1(0.3)+2(0.3)+3(0.2)+4(0.1)$$
$$= 1.9$$

The variance of x is defined as $Var(X) = E[(X-u)]^2 = \sum (X-u)^2 P(X)$

$$= (0-1.9)^2(0.1) + ...+ (4-0.9)^2(0.1)$$
$$= 1.29$$

The standard deviation is 1.136.

d) Using the table form of the probability distribution given in the exercise,

$P(X>2) = 0.2+0.1 = 0.3$

e) $P(X \leq 3) = 1-P(X = 4) = 1-0.1 = 0.9$

4.87

a-b On the first try, the prob of selecting the proper key is ¼. If the key is not found on the first try, the probability changes on the second try. Let F denote a failure to find the key and S denote a success. The random variable is x, the number of keys tried before the correct key is found. The four associated simple events are shown below:

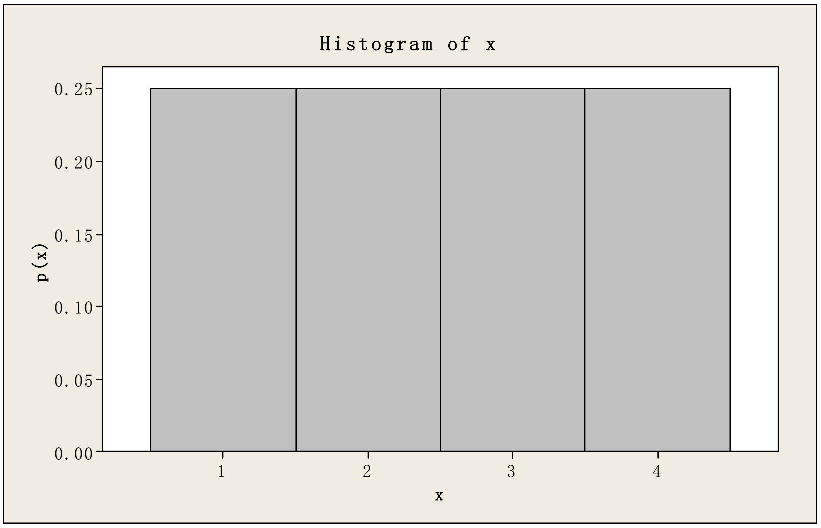
E1: S (X=1)
E2: FS (X=2)
E3: FFS (X=3)
E4: FFFS (X=4)

c-d Then P(1)= P (X=1) = P(s) =1/4

P (2) = P (X=2) = P (FS) = P(F)P(S) = (3/4) (1/3) = 1/4
P (3) = P(X=3) = P (FFS) = P(F)P(F)P(S) = (3/4) (2/3) (1/2) = 1/4
P (4) = P(X=4) = P (FFFS) = P(F)P(F)P(F)P(S) = (3/4)(2/3)(1/2)(1) = 1/4

The probability distribution and probability histogram follow:

X	1	2	3	4
P(X)	1/4	1/4	1/4	1/4



4.96

a) By definition: $E(X) = \sum X\, P(X) = 3(0.03) + 4(0.05) + ... + 13(0.01) = 7.9$

b) $Var(X) = \sum (X-u)^2 P(X) = (3-7.9)^2(0.03) + ... + (13-7.9)^2(0.01) = 4.73$

so standard deviation = $\sqrt{4.73} = 2.174856$

c) $u + 2\sigma = 7.9 + 4.349712 = (3.55, 12.25)$, so 1-P(x=3)-P(x=13) =1-0.03-0.01 =0.96 ,so approximately 96% of all measurements falls into the interval $u+2\sigma$.

5.24

(0.15) (0.15) = 0.0225

5.25

Define x to be the number of cars that are black, then P= P[black] = 0.1 and n= 25

Use Table 1 in Appendix I:

a) $P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.902 = 0.098$

b) $P(X \geq 6) = 0.991$

c) $P(X > 4) = 1 - P(X \leq 4) = 1 - 0.902 = 0.098$

d) $P(X=4) = P(X \leq 4) - P(X \leq 3) = 0.902 - 0.764 = 0.138$

e) $P(3 < X < 5) = P(X \leq 5) - P(X \leq 2) = 0.967 - 0.537 = 0.430$

f) P (more than 20 not black) = P(less than 5 black) = $P(X \leq 4) = 0.902$

5.43

Let X to be the number of misses during a given month.

Then x has a Poisson distribution with u=5

a) $P(0) = e^{-5} = 0.0067$

b) $P(5) = 5^5 e^{-5}/5! = 0.1755$

c) $P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.440 = 0.560$ from Table 2.

5.48
By Definition of the Poisson Probability Distribution:
$P(X=x) = \lambda^x e^{-\lambda} / X!$ x=0,1,2,,,,
We're given E.coli have occurred in Colorado at a rate of 2.5 per 10,000 for a period of 2 years, so for part (a), we have $\lambda = 2.5/2 = 1.25$

a) $P(X \le 5) = P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$
$= 0.2865048 + 0.358131 + 0.2238319 + 0.09326328 + 0.02914478 + 0.007286194$
$= 0.998162$

b) $P(X>5) = 1 - P(X \le 5) = 1 - 0.998162 = 0.001838$

c) So approximately 95% of occurrences of E. coli involve at most 3 cases with $\lambda = 1.25$.

5.52
For this question, N= 8, n=3, M=5 blue candies and N-M=3 red candies, so follow the hypergeometic distribution formula, we can get $P(X=x) = C_x^5 C_{3-x}^3 / C_3^8$

a) $P(X=2) = C_2^5 C_1^3 / C_3^8 = 15/28 = 0.5357$

b) P(X=0) = P (the candies are all red) = $C_0^5 C_3^3 / C_3^8 = 0.01786$

c) $P(X=3) = C_3^5 C_0^3 / C_3^8 = 0.1786$

5.56
We are given N=10, n=4, M=5, N-M=5 and $P(X) = C_x^5 C_{4-x}^5 / C_4^{10}$

a) $P(X=4) = C_4^5 C_0^5 / C_4^{10} = 0.0238$

b) $P(X \le 3) = P(X=0)+P(X=1)+P(X=2)+P(X=3)=0.976$

c) $P(2 \le X \le 3) = 0.3296703 + 0.07326007 = 0.714$

Q10.
Expressions for the mean and variance of Binomial distribution:

$$E(x) = \sum_{x=0}^{n} x P\{x = x\}$$

$$= \sum_{x=0}^{n} x \; {}^nC_x \, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} \frac{x \, n!}{x! \, (n-x)i} p^x q^{n-x}$$

$$= np \sum_{x=0}^{n-1} \frac{(n-1)!}{(x-1)! \, \{(n-1)-(x-1)\}!} p^{x-1} q^{(n-1)-(x-1)}$$

$$= np \sum_{x=0}^{n-1} {}^{n-1}C_x \; p^{x-1} \; q^{n-x}$$

$$= np(p+q)^{n-1}$$

$$E(x) = np \qquad (\because p + q = 1)$$

To find the variance:

$$E(x^2) = \sum_{x=0}^{n} x^2 p\{X = x\}$$

$$= \sum_{x=0}^{n} (x(x-1)+x)\, p(X = x)$$

$$= \sum_{x=0}^{n} [x(x-1)]P(X = x) + \sum_{x=0}^{n} xP(X = x)$$

$$= \sum_{x=0}^{n} \frac{x(x-1)n!}{x!\,(n-x)!}\, p^x q^{n-x} + np$$

$$= \sum_{x=2}^{n} \frac{n(n-1(n-2)!}{(x-2)!(n-x)!} p^{x-2+2}(1-q)^{(n-2)-(x-2)} + np$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2}(1-p)^{(n-2)-(x-2)} + np$$

Let y=x-2

$$= n(n-1)p^2 \sum_{y=0}^{n-2} \frac{(n-2)!}{y!(n-2-y)!}\, p^y (1-p)^{n-2-y} + np$$

$$= n(n-1)p^2(p+q)^{n-2} + np$$

$$= n^2p^2 - np^2 + np$$

$$= n^2p^2 + np(1-p)$$

$$= n^2p^2 + npq$$

Now, $V(X) = E(x^2) - [E(x)]^2$

$$= n^2p^2 + npq - n^2p^2$$

$$= npq$$

Expressions for the mean and variance of Poisson distribution:

$$E(x) = \sum_{x=0}^{\infty} xP\{X = x\}$$

$$= \sum_{x=0}^{\infty} x.\frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \lambda \sum_{x=1}^{\infty} \frac{e^{-\lambda}\lambda^{x-1}}{(x-1)!}$$

$$= \lambda e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}$$

$$= \lambda e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda^y}{y!} \quad \text{(Put } y = x - 1\text{)}$$

$$= \lambda e^{-\lambda} e^{\lambda}$$

$$= \lambda$$

$$E(X^2) = \sum_{x=0}^{\infty} x^2 P\{X = x\}$$

$$= \sum_{x=0}^{\infty} x^2 . \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= \sum_{x=0}^{\infty} \{x(x-1) + x\} . \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= \sum_{x=0}^{\infty} \frac{x(x-1)e^{-\lambda}\lambda^x}{x!} + \sum_{x=0}^{\infty} \frac{x\ e^{-\lambda}\lambda^x}{x!}$$

$$= \sum_{x=2}^{\infty} \frac{e^{-\lambda} \lambda^x}{(x-2)!} + \sum_{x=1}^{\infty} \frac{e^{-\lambda} \lambda^x}{(x-1)!}$$

$$= \lambda^2 \sum_{x=2}^{\infty} \frac{e^{\lambda} \lambda^{x-2}}{(x-2)!} + \lambda \sum_{x=1}^{\infty} \frac{e^{-\lambda} \lambda^{x-1}}{(x-1)!}$$

$$= \lambda^2 e^{-\lambda} e^{\lambda} + \lambda e^{-\lambda} e^{\lambda}$$

$$= \lambda^2 + \lambda$$

$$V(X) = E(x^2) - \{E(X)\}^2 = \lambda^2 + \lambda - \lambda^2 = \lambda$$