

#1

$$ME = 1.96 \cdot \sqrt{\frac{pq}{n}}$$

(since margin of error = 0.02)

a) The hypothesis testing is:

$$(\alpha = 0.05, Z_{\alpha} = 1.96)$$

$$0.02 = 1.96 \cdot \sqrt{\frac{0.4(0.6)}{n}}$$

$$H_0: p = 0.5 \quad H_a: p > 0.6$$

$$\left(\frac{0.02}{1.96}\right)^2 = \frac{0.4(0.6)}{n}$$

b) Level of Significance = $\Pr\{X \geq 17 | H_0: p = 0.6\}$

$$n = \frac{0.4(0.6)}{\left(\frac{0.02}{1.96}\right)^2} \approx 2305$$

Therefore, the minimum sample size necessary to satisfy her goal is 2305.

b) If the prior information is not available, $p = 0.5$ to be conservative

$$ME = 1.96 \cdot \sqrt{\frac{pq}{n}}$$

c) The power of a statistical test is $1 - \beta$.

$$0.02 = 1.96 \cdot \sqrt{\frac{0.5(1-0.5)}{n}}$$

$1 - \beta = \Pr\{\text{reject } H_0 \text{ when } H_a \text{ is true}\}$

$$\left(\frac{0.02}{1.96}\right)^2 = \frac{0.25}{n}$$

$$1 - \beta = \Pr\{X \geq 16 | p = 0.8\}$$

$$n \approx 2401$$

$$= 1 - \Pr\{X \leq 16\}$$

Therefore, the minimum sample size would be 2401.

$$1 - \beta = \Pr\{X \geq 16 | p = 0.8\}$$

$$= 1 - \Pr\{X \leq 15\}$$

$$= 1 - 0.370 = 0.630$$

d) The second power value (when $X \geq 16$) is larger than the first one.

If α is increased, the β decreases, and vice versa.

The only way to decrease β for a fixed α is to "buy" more information that is, increase the sample size.