

Assignment 7/ Solution

7.28 a) If a random sample of n measurements is selected from a population with mean μ and standard deviation σ , the sample distribution of the sample mean \bar{x} will have mean μ and standard deviation σ / \sqrt{n} .

b) From the Empirical Rule (and the general properties of the normal distribution) approximately 95% of the measurements will lie in 2 standard deviations of the mean:

$$\mu \pm 2\sigma / \sqrt{n} = (64575, 66640).$$

$$c) p(\bar{x} > 67000) = p\left(z > \frac{67000 - 65608}{4000 / \sqrt{n}}\right) = 0.0035$$

d) \$67000 does not lie in the interval (64575, 66640), so we would consider this is unusual.

7.30 a) The approximate sampling distribution of the sample mean of $n=10$ is:

$$\bar{x} \sim N(20, 2^2 / 10).$$

$$b) p(\bar{x} < 20) = p\left(z < \frac{20 - 21}{2 / \sqrt{20}}\right) = p(z < -1.58) = 0.0571$$

$$c) p(\bar{x} < 20) = 0.001 \Rightarrow p\left(z < \frac{20 - \mu}{2 / \sqrt{20}}\right) = 0.001$$

$$\Rightarrow p(z < -3.1) = 0.001 \Rightarrow \mu = 21.95$$

7.44 a) $\hat{p} = 66\%$ since $n\hat{p} = 660$ and $n\hat{q} = 340$, both of them are greater than 5, the binomial distribution can be approximately by a normal distribution with mean $p=0.66$ and $SE=0.015$.

$$b) z = \frac{\hat{p} - p}{\sqrt{pq/n}} \approx 1.33, \text{ so } p(z > 1.33) = 0.09$$

$$c) z = \frac{\hat{p} - p}{\sqrt{pq/n}} \approx -1.33, \text{ so } p(-1.33 < z < 1.33) = 0.8164$$

$$d) z = \frac{0.7 - 0.66}{0.0498} = 2.67, \text{ then } p(\hat{p} > 0.7) = 1 - p(\hat{p} < 0.7) = 1 - p(z < 2.67) \\ = 1 - 0.9962 = 0.0038$$

$$7.46 a) z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.25 - 0.15}{0.036} = 2.78, p(\hat{p} > 0.25) \approx p(z > 2.78) = 0.0027$$

$$b) z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.12 - 0.15}{0.036} = -0.83, \quad p(\hat{p} < 0.12) \approx p(z < -0.84) = 0.2005$$

$$c) z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.3 - 0.15}{0.036} = 4.16, \quad \text{the value is unusual because } \hat{p} = 0.3$$

lies 4.16 standard deviations above the mean $p=0.15$.

7.57

