9.2 P-value Approach 2

Test Stat.	Significance Level	One or Two-tailed test	p-value	p-value < α?	Conclusion
$z=3.01$	$\alpha=0.05$	Two tailed	0.0026	Yes	reject Ho
$z=2.47$	$\alpha=0.05$	one tailed (upper)	0.0068	Yes	reject Ho
$z=-1.30$	$\alpha=0.01$	Two tailed	0.1936	No	Do not reject Ho.
$z=-2.88$	$\alpha=0.01$	one tailed (lower)	0.0020	Yes	reject Ho.

9.6 In this question, the parameter of interest is μ, the population mean. The objective of the experiment is to show that the mean exceed 2.3.

a) We want to prove the alternative hypothesis that μ is, in fact, greater than 2.3. Hence, the alternative hypothesis is Ha: $\mu > 2.3$ and the null hypothesis is Ho: $\mu = 2.3$.

b) The best estimator for μ is the sample average $\bar{x}$, and the test stat is

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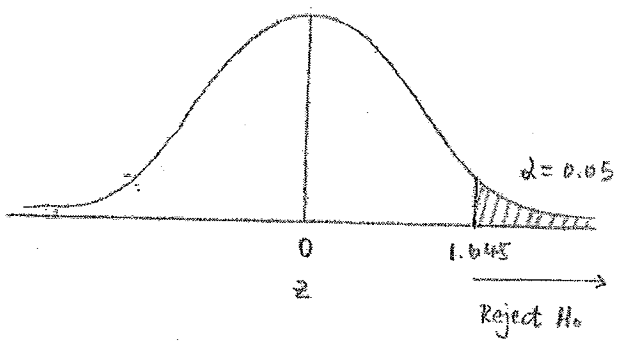
$$z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$$

which represents the distance (measured in units of standard deviations) from $\bar{x}$ to the hypothesized mean μ. Hence, if this value is large in absolute value, one of two conclusions may be drawn. Either a very unlikely event has occurred, or the hypothesized mean is incorrect. Refer to part a. If $\alpha = 0.05$, the critical value of z that separate the rejection and non-rejection regions will be a value (denote by z_0) such that

$$p(Z > z_0) = \alpha = 0.05$$

That is, $z_0 = 1.645$ (see below). Hence, Ho will be rejected if $z > 1.645$.



The standard error of the mean is found using the sample standard deviation to approximate the population standard deviation σ:

$$SE = \frac{\sigma}{\sqrt{n}} \approx \frac{s}{\sqrt{n}} = \frac{0.29}{\sqrt{35}} = \boxed{0.049}$$

) To conduct the test, calculate the value of the test statistic using the information contained in the sample. Note that the value of the true standard deviation, σ, is approximated using the sample standard deviation s.

$$z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \approx \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{2.4-2.3}{0.049} = \boxed{2.04}$$

The observed value of the test statistic, $z = 2.04$, falls in the rejection region and the null hypothesis is rejected. There is sufficient evidence to indicate that $\mu > 2.3$.

9.7

Refer to Exercise 9.6.

a) $p\text{-value} = P(z > 2.04) = 1 - 0.9793 = \boxed{0.0207}$

b) Since $p\text{-value} = 0.0207 < \alpha = 0.05$, so we reject H_0

c) Compare the conclusion in part b) with the conclusion reached in part d) of exercise 9.6, they are same, we reject the null hypothesis.

9.11

a) In order to make sure the average weight was one pound, we should test

$H_0: \mu = 1$ vs $H_a: \mu \neq 1$

b-c) The test statistic is:

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$$z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \approx \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{1.01-1}{0.18/\sqrt{35}} = 0.33$$

with $p\text{-value} = P(|z| > 0.33) = 2\cdot(0.3707) = \boxed{0.7414}$ since the p-value is greater than 0.05, the null hypothesis should not be rejected. The manager should report that there is insufficient evidence to indicate that the mean is different from 1.

9.28

a) Given

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Control	Experimental
$n_1 = 30$	$n_2 = 40$
$\bar{x}_1 = 15$	$\bar{x}_2 = 23$
$s_1 = 4$	$s_2 = 10$

we want to test $H_0: \mu_1 = \mu_2$ vs $H_a: \mu_1 < \mu_2$

substituting into the formula for the test statistic, you get:

$$z \approx \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(15-23)}{\sqrt{\frac{4^2}{30} + \frac{16^2}{40}}} = \frac{-8}{\sqrt{3.03}} \approx -4.60$$

$\alpha = 0.01$, $z_\alpha = z_{0.01} = 2.33$

Since $z = -4.60 < -z_\alpha = -2.33 \Longrightarrow$ So, we reject H_0 yes, there is sufficient evidence to indicate that the average time to complete the task was longer for the experimental "rock music" group.

b)

$$(\bar{X}_1 - \bar{X}_2) + 2.33 \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = -8 + 2.33 \cdot \sqrt{3.03} = -3.95$$

Since the interval $(-\infty, -3.95]$ does not contains the value of the parameter specified by H_0, so we reject H_0, so this interval confirm my conclusion in part a).

9.35 a) $H_0: p = 2/3$ b) $H_a: p > 2/3$

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c) $\hat{p} = \frac{x}{n} = \frac{164}{200} = 0.82$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.82 - 0.67}{\sqrt{(2/9)/200}} = 4.6$$, $\alpha = 0.05$, $z_\alpha = 1.645$

Since $z = 4.6 > z_\alpha = 1.645$, we reject H_0.

d) p-value $= P(z > 4.6) = 1 - P(z < 4.6) \approx 0$, since p-value is smaller than 0.05 so we reject H_0 which confirm the conclusion from part c).

9.48

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placebo	Prempro
$x_1 = 21$	$x_2 = 40$
$n_1 = 2266$	$n_2 = 2266$

$H_0: p_1 = p_2$ vs $H_a: p_1 < p_2$,

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{-0.0084}{0.031} = -2.45$$, Since $\alpha = 0.01$, $z_\alpha = 2.33$, so $z < -z_\alpha = -2.33$, so we do reject H_0. that means there is not suffient evidence to indicate that the risk of dementia is higher for patients using Prempro

and $\hat{p}_1 = \frac{21}{2266} = 0.0093$, $\hat{p}_2 = \frac{40}{2266} = 0.0177$, $\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = 0.013$