Solution / Arts & Science 2R06 Test 3

1) We're given mean=16 mm Hg and standard deviation = 3 mm Hg

a(1 mark): P($12 \le X \le 20$) = P ($\frac{12-16}{3} \le X \le \frac{20-16}{3}$)

= P ($-1.33 \le X \le 1.33$)

= 0.9082-0.0918

=0.8164

So, 81.64% of the general population would fall within this range.

b(1 mark): P ($X \ge 22$)= P($\frac{X-\mu}{\sigma} \ge \frac{22-16}{3}$) =P($Z \ge 2$)=1-0.9772 =0.0228

So, 2.28% of the population would have their intraocular pressure to be more than 22 mm Hg.

c(2 marks): 2.5SD= 2.5(3)=7.5, we want to find the percentage would have their intraocular pressure to be at least 2.5 standard deviation away from the mean, ie:

P ($16-7.5 \le X \le 16+7.5$) = P ($8.5 \le X \le 23.5$) = P ($\frac{8.5-16}{3} \le z \le \frac{23.5-16}{3}$)

= P ($-2.5 \le Z \le 2.5$)

= 0.9938-0.0062

=0.9876

So, 98.76% of the population would have their intraocular pressure to be at east 2.5 standard deviation away from the mean.

2) We're given p=0.05, n=500, we get np=25 and nq= 475, so we could use the normal approximation to the binomial probabilities will be adequate since np>5 and nq> 5.

a(1 mark):

$$P(X \ge 27) \approx P(Z \ge \frac{26.5-25}{\sqrt{npq}})$$

$$= P(Z \ge \frac{1.5}{4.873}) = 1-0.6217$$

$$= 0.3783$$

b(1 mark):

$$P(15 \le X \le 22) \approx P(\frac{14.5-np}{4.873} \le Z \le \frac{22.5-np}{4.873})$$

$$= P(\frac{-10.5}{4.873} \le Z \le \frac{-2.5}{4.873})$$

$$= P(-2.154 \le Z \le -0.513) = 0.2892$$

c(2 marks):

$$P(X = 29) = P(X \le 29) - P(X \le 28)$$

$$= P(Z \le \frac{29.5-25}{4.873}) - P(Z \le \frac{28.5-25}{4.873})$$

$$= 0.9212 - 0.7642$$

$$= 0.057$$

3) we're given mean=7.7 , variance = σ^2 = 4.41, n=16

a(2 marks):

$$P(7 \le \bar{X} \le 8) = P(\frac{7-7.7}{2.1/\sqrt{16}} \le Z \le \frac{8-7.7}{2.1/\sqrt{16}})$$
$$= P(-1.33 \le Z \le 0.57) = 0.7157 - 0.0918$$
$$= 0.6239$$

b(2 marks):

$$P(\bar{X} > 9) = P(Z > \frac{9-7.7}{2.1/\sqrt{16}})$$
$$= P(Z > 2.48) = 1 - 0.9934 = 0.0066$$

4) Margin of error=0.02, we know ME = $1.96\sqrt{\frac{pq}{n}}$, p =0.5 (conservative estimate.)

a(2 marks): $0.02 = 1.96\sqrt{\frac{0.5(1-0.5)}{n}}$ =>0.010204= $\sqrt{\frac{0.25}{n}}$ =>n=2401

b(2 marks): $0.01 = 1.96\sqrt{\frac{0.5(1-0.5)}{n}}$ =>0.0051204= $\sqrt{\frac{0.25}{n}}$ =>n=9604

Both a) and b) require 95% confidence interval and therefore the one way to decrease the margin of error is to increase the sample population size. So we can improve the accuracy of our estimate either decrease confidence interval or increase the sample population size n.

5)

	Group 1 (oc user)	Group 2 (non-oc users)
mean	132.86	127.44
Standard deviation	15.34	18.23
Sample size	8	22

a(3 marks): 95% CI is:

$$(\bar{X}_1 - \bar{X}_2) \pm 1.96\sqrt{\frac{S_1^{\ 2}}{n_1} + \frac{S_2^{\ 2}}{n_2}}$$

$$= (132.86 - 127.44) \pm 1.96\sqrt{\frac{15.34^2}{8} + \frac{18.23^2}{22}}$$

= $(-7.66, 18.498)$

b(1 mark): Based on the statistical evidence there is not a significant difference in the mean blood pressure between the two groups as 0 is included in the interval.