#10.9 a) Similar to previous exercises. The hypothesis to be tested is:

$$H_0: \mu = 100 \quad vs \quad H_a: \mu < 100$$

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$$\bar{x} = \frac{\sum x_i}{n} = \frac{1797.095}{20} = 89.85475$$

$$s^2 = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1} = \frac{165697.7081 - \frac{(1797.095)^2}{20}}{19} = 222.1150605 \text{ and } s = 14.9035$$

The test statistic is:

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{89.85475 - 100}{14.9035/\sqrt{20}} = -3.044$$

The critical value of t with $\alpha = 0.01$ and $n-1 = 19$ degrees of freedom is $t_{.01} = 2.539$ and the rejection region is $t < -2.539$. The null hypothesi is rejected and we conclude that μ is less than 100DL.

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b) The 99% upper one-sided confidence bound, based on $n-1=19$ degrees of freeom, is

$$\bar{x} + t_{.01} \cdot \frac{s}{\sqrt{n}} \Rightarrow 89.85475 + 2.539 \cdot \frac{14.9035254}{\sqrt{20}}$$

$$\Rightarrow \mu < 98.316$$

This confirms the results of part a) in which we concluded that th mean is less than 100 DL

#10.6

Solution / Assignment 10

1 a) $\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} = 0.8964 \pm 2.16 \cdot \frac{0.3995}{\sqrt{14}} = (0.6658\ ,\ 1.127)$

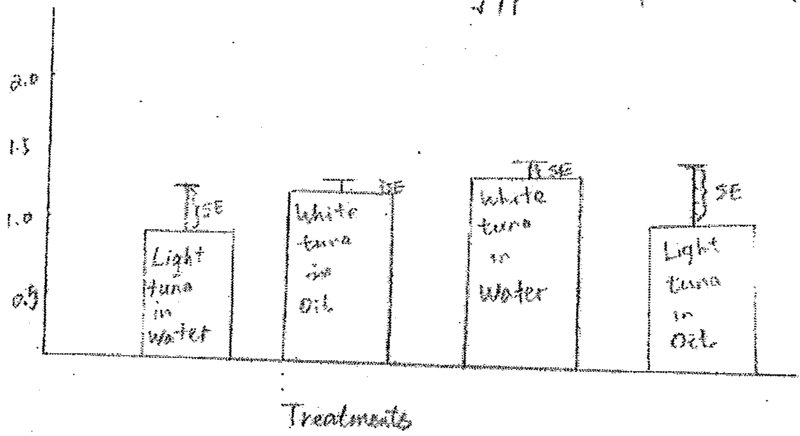
That means we have 95% for the average price for light tuna in water within this interval (0.6658, 1.127).

1 b) $\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} = 1.225 \pm 3.182 \cdot \frac{0.033}{\sqrt{4}} = (1.172\ ,\ 1.278)$

This confidence interval is much smaller interval width than above.

1 c) $\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} = 1.28 \pm 2.365 \cdot \frac{0.135119}{\sqrt{8}} = 1.28 \pm 2.365(0.048) = (1.166, 1.394) \rightarrow$ white tuna in water

$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} = 1.147 \pm 2.228 \cdot \frac{0.6785}{\sqrt{11}} = 1.147 \pm 2.228(0.2046) = (0.69\ ,\ 1.60) \rightarrow$ Light tuna in oil



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#10.24

1 a) $H_0: \mu_1 = \mu_2$ vs $H_a: \mu_1 < \mu_2$.

b) we can calculate the test statistic :

$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2(\frac{1}{n_1} + \frac{1}{n_2})}}$, the first thing we should get $S^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$

2 we're given (Antiplaque Group) $n_1 = 7$, $\bar{x}_1 = 0.78$, $S_1 = 0.32$

(Control Group) $n_2 = 7$, $\bar{x}_2 = 1.26$, $S_2 = 0.32$

$$\Rightarrow S^2 = \frac{6 \cdot 0.32^2 + 6 \cdot 0.32^2}{7+7-2} = \frac{1.2288}{12} = 0.1024$$

$$t = \frac{0.78 - 1.26}{\sqrt{0.1024 \cdot (\frac{1}{7} + \frac{1}{7})}} = \frac{-0.48}{0.1710472} = -2.806$$

Since $-t_\alpha = -t_{0.05} = -1.782$, and $t = -2.806 < -t_{0.05} = -1.782$

so we need to reject H_0, that means the data provide sufficient evidence to indicate that the oral antiplaque rinse is effective.

c) Since the observed value, $t = -2.806$, lies between $t_{0.01} = -2.681$ and $t_{0.005} = -3.05$

2 the tail area to the left of -2.806 is between 0.005 and 0.01

The p-value for this test would be reported as

$0.005 <$ p-value < 0.01

10.26

a) Test Hypothese $H_0: \mu_1 = \mu_2$ vs $H_a: \mu_1 \neq \mu_2$

we're given $n_1 = 10$, $\bar{x}_1 = 14.5$, $S_1 = 3.92$ and $\alpha = 0.05$

$n_2 = 10$, $\bar{x}_2 = 11.1$, $S_2 = 3.98$

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$$S^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = \frac{9 \cdot 3.92^2 + 9 \cdot 3.98^2}{10+10-2} = \frac{280.8612}{18} = 15.6034$$

$$t = \frac{14.5 - 11.1}{\sqrt{15.6034 \cdot (\frac{1}{10} + \frac{1}{10})}} = 1.925$$

Since $t_{18, 0.025} = 2.101$ and $t < t_{\alpha/2}$, so we do not reject H_0

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b) $S^2 = \frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}$ under the condition of exercising at 80% of maximal oxygen consumption.

$= \frac{9\cdot 3.49^2+9\cdot 4.95^2}{16}$

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$= \frac{330.1034}{16} = 20.63$

95% CI: $(\bar{x}_1-\bar{x}_2) \pm t_{\alpha/2}\cdot\sqrt{20.63(\frac{1}{10}+\frac{1}{10})} = (-3.324, 4.724)$

c) we use the unpooled t-test. Since the ratio of the two sample varian[ces]
$\frac{\text{Larger } S^2}{\text{Smaller } S^2} = \frac{16.9^2}{4.67^2} > 3$, and the population variances are not equal, the

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pooled estimator S^2 is no longer approximate, and each population variance mus[t] be estimated by its corresponding sample variance. The test statistic is:

$$t = \frac{\bar{x}_1-\bar{x}_2}{\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}} \quad \text{with } df = \frac{\left(\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}\right)^2}{\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{S_2^2}{n_2}\right)^2}{n_2-1}}$$

10.37

a) It is necessary to use a paired-difference test. Since the two samples are not random and independent. The hypothesis of interest is

$H_0: \mu_1-\mu_2=0$ or $H_0: \mu_d=0$

$H_a: \mu_1-\mu_2\neq 0$ or $H_a: \mu_d\neq 0$

The table of differences, along with the calculation of $\bar{d}$ and S_d^2, is presented be[low]

d_i	0.1	0.1	0	0.2	-0.1	$\Sigma d_i = 0.3$
d_i^2	0.01	0.01	0.00	0.04	0.01	$\Sigma d_i^2 = 0.07$

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$\bar{d} = \Sigma d_i/n = 0.3/5 = 0.06$ and $S_d^2 = \frac{\Sigma d_i^2 - \frac{(\Sigma d_i)^2}{n}}{n-1} = \frac{0.07-\frac{(0.3)^2}{5}}{4} = 0.013$

The test statistic is:

$$t = \frac{\bar{d}-\mu_d}{S_d/\sqrt{n}} = \frac{0.06-0}{\sqrt{\frac{0.13}{5}}} = 1.177$$

With $n-1=4$ degrees of freedom. The rejection region with $\alpha=0.05$ is $|t| > t_{.025} = 2.776$, and H_0 is not rejected. We cannot conclude that the means are different.

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1 b) p-value is: $p(|t|>1.177) = 2P(t>1.177) > 2(0.10) = 0.20$

c) A 95% confidence interval for $\mu_1 - \mu_2 = \mu_d$ is:

1 $\bar{d} \pm t_{.025} \cdot \frac{S_d}{\sqrt{n}} \Rightarrow 0.06 \pm 2.776 \cdot \sqrt{\frac{0.013}{5}} \Rightarrow 0.06 \pm 0.142 = (-0.082, 0.202)$

1 d) In order to use the paired-difference test, it is necessary that the n paired observation be randomly selected from normally distributed populations.

10.61 The hypothesis of interest is $H_0: \sigma_1^2 = \sigma_2^2$ vs $H_a: \sigma_1^2 \neq \sigma_2^2$ and the test sta is $F = \frac{S_1^2}{S_2^2} = 71^2/69^2 = 1.059$

The critical value of F for various value of α are given below using $df_1 = 15$ and $df_2 = 14$

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α	0.10	0.05	0.025	0.01	0.005
F_α	2.01	2.46	2.95	3.66	4.25

Hence,

$p\text{-value} = 2P(F > 1.059) > 2(0.10) = 0.20$

Since the p-value is so large. Ho is not rejected. There is no evidence to indicate that the variance are different

11.10 a) The following preliminary calculations are necessary:

$T_1 = 380 \quad T_2 = 199 \quad T_3 = 261 \quad G = 840$

$CM = \frac{(\Sigma X_{ij})^2}{n} = \frac{(840)^2}{11} = 64{,}145.4545$

Total $SS = \Sigma X_{ij}^2 - CM = 65{,}286 - CM = 1140.5455$

$SST = \Sigma \frac{T_i^2}{n_i} - CM = \frac{380^2}{5} + \frac{199^2}{3} + \frac{261^2}{3} - CM = 641.87883$

3 Calculate $MS = SS/df$ and consolidate the information in an ANOVA table:

Source	df	SS	MS
Treatments	2	641.8788	320.939
Error	8	498.6667	62.333
Total	10	1140.5455	

b) The hypothesis to be tested is:

$H_0: \mu_1 = \mu_2 = \mu_3$ vs H_a: at least one pair of means are different

and the F test to detect a difference in mean student response is

3 $$F = \frac{MST}{MSE} = 5.15$$

The rejection region with $\alpha = 0.05$ and 2 and 8 df is $F > 4.46$ and H_0 is rejected. There is a significant difference in mean response due to the three difference methods.

11.35

1 a) 7 Blocks

1 b) 7 observations in each treatment in total

1 c) 5 observations in each blocks in total

1 d)

	DF	SS	MS	F
Treatments	4	14.2	3.55	9.673
Blocks	6	18.9	3.15	8.58
Errors	24	8.8	0.367	
Total	34	41.9		

1 e) $F = 9.67$, $F_{4,24,0.05} = 2.78$, since $9.67 > F_\alpha \Rightarrow$ reject H_0

There is sufficient evidence to indicate differences among the treatment means.

1 f) $F = 8.58$, $F_{6,24,0.05} = 2.51$. Since $8.58 > F_\alpha \Rightarrow$ reject H_0

There is sufficient evidence to indicate differences among the block means.