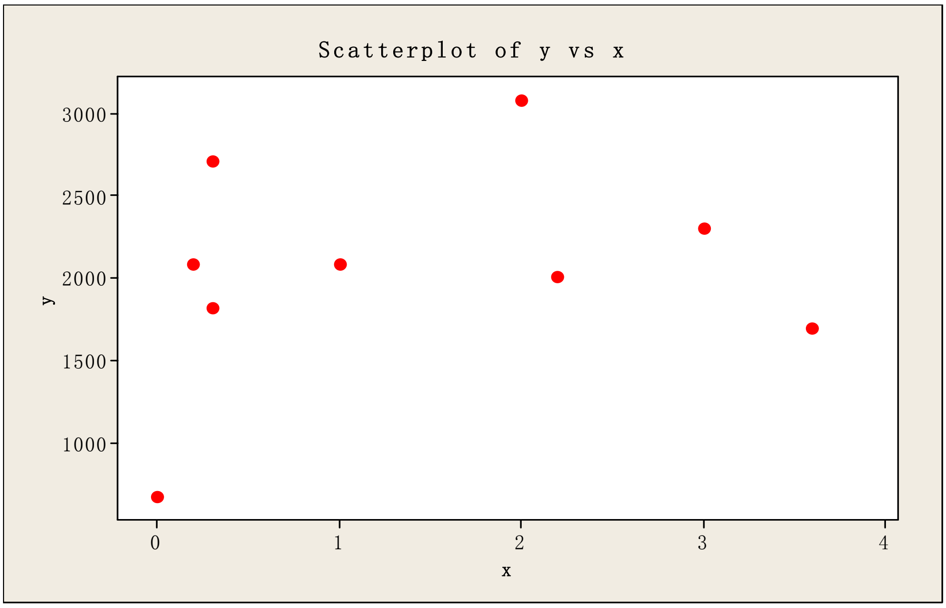
Question 1:

a) Scatter plot:



b) The points on the scatter plot do not have a strong sloped pattern, it seems to be a very leak correlation between these two things, as there is no clear trend in the plot.

c) r= Sxy/ SxSy = 0.22 and which agrees with part (b) since the r value is very small (very weak correlation).

d) By definition, b= r(Sy/Sx) = 112.1 and a= $\bar{y}$ -b $\bar{x}$ =1894.8

and the least-squares line is y= 1894.8+112.1x

e) when x= 2.9%, y= 2219.89

Question2:

Line	Rework (R)	Produced
A1	0.05	0.5
A2	0.08	0.3
A3	0.1	0.2

a) Let R be the probability that a component needs rework.

P (R) = (0.05)(0.5)+(0.08)(0.3)+(0.1)(0.2) = 0.069

b) By Bayes' Rule: P(A1/R)= $\dfrac{P(\mathrm{A1} \cap R)}{P(R)}$ =(0.5)(0.05)/ 0.069 =0.362

c) P(A2/ R)= $\dfrac{P(\mathrm{A2} \cap R)}{P(R)}$ = (0.3)(0.08)/ 0.069 =0.348

d) P(A3/ R) = $\dfrac{P(\mathrm{A3} \cap R)}{P(R)}$ = (0.2)(0.1)/0.069 =0.2898

Question3:

Binomial Distribution: P(X=x) = $\sum_{x=0}^{n} C_x^n (p)^x (q)^{n-x}$

a) Based on question, we consider to reject the claim that p$\geq$0.8 whenever x$\leq$14

That means P(x$\leq$14) = $\sum_{x=0}^{14} C_x^{20}(0.8)^x(0.2)^{20-x}$ = 0.196 (Since there is a Table 1 available for n=20 and p=0.8), so we should use it rather than the binomial formula to calculate the necessary probabilities.)

b) P(Rejecting) = $\sum_{x=0}^{14} C_x^{20}(0.7)^x(0.3)^{20-x}$ = 0.584 (by checking Table 1 for n-20 and p=0.7)

P(not Rejecting) =1- P(Rejecting) = 1-0.584 =0.416

c) If the value x 14 is replaced by13, P (Rejecting)= 0.087 with n=20 and p= 0.8
and P(not Rejecting) = 0.608 with n=20 and p= 0.7.
Therefore, we can conclude the part (a) error probabilities go decreasing and part (b) error probabilities go increasing.

Question 4:

a) Since n is large and λ = np = 400 (0.005) =2 (which is smaller than 7), so we use the Poisson approximation to the binomial distribution .

P(X=1) = $P(X=1) = \frac{e^{-2}2^1}{1!}$ = 0.271

b) P(X$\leq$3) = P(X=0)+P(X=1)+P(X=2) = 0.857 (by checking the Table 2).

Question5:

a) P(X=x) = $\dfrac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$

b) E(X) = $\sum_{x=0}^{n} xp(X=x)$

$$\sum_{x=0}^{n} xp(X=x) = \sum_{x=0}^{n} x\frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}} = \sum_{x=1}^{n}\frac{M!}{(x-1)!(M-x)!}\frac{(N-M)!}{(n-x)!(N-M-n-x)!}\frac{n!(N-n)!}{N!}$$

$$=\sum_{x=1}^{n}\frac{M(M-1)!(N-M)!}{(x-1)!(M-x)!(n-x)!(N-M-n-x)!}\frac{n(n-1)!(N-n)!}{N(N-1)!}$$, then let y=x-1

$$=\frac{nM}{N}\sum_{y=0}^{n-1}\frac{(M-1)!(N-M)!}{y!(M-1-y)!(n-1-y)!(N-M-n-1-y)!}\frac{1}{\binom{N-1}{n-1}}$$

$$= \frac{nM}{N}\sum_{y=0}^{n-1}\frac{\binom{M-1}{y}\binom{(N-1)-(M-1)}{n-1-y}}{\binom{N-1}{n-1}} = \frac{nM}{N}\sum_{y=0}^{n-1}\frac{\binom{M-1}{y}\binom{N-M}{n-1-y}}{\binom{N-1}{n-1}}$$

(then we substitute x for y)

$$= \frac{nM}{N}\sum_{x=0}^{n-1}\frac{\binom{M-1}{x}\binom{N-M}{n-1-x}}{\binom{N-1}{n-1}}$$

$$= \frac{nM}{N}$$