<u>Arts/ Sci 2R06 Assignment 6 Solution</u>

6.14: We are given $P(Z>7.5) = 0.8023$ and standard deviation is equal to 2,
so we know $P(Z<7.5) = 0.1977$
By checking the Table 3, we can find Z=-0.85, since $Z = (X-\mu)/\sigma$,
then $\mu = 7.5+0.85(2) = 9.2$

6.17 Since $P(X>4)=0.9772$,so $P(X \leq 4)=1-0.9772= 0.0228$
$P(X>5)=0.9332$, so $P(X \leq 5) = 1-0.9332 = 0.0668$
By checking Table 3, we know $P(Z=-2) = 0.0228$ and $P(Z=-1.5) = 0.0668$
So, we can solve these two equations with two unknowns,
$-2 = (4-\mu)/\sigma$ and $-1.5 = (5-\mu)/\sigma$
Therefore, $\sigma = 2$, $\mu = 8$.

6.18 It is given that the random variable X (the weights of these "1-pound"package)are normally distributed with $\mu = 1$, $\sigma = 0.15$.

a) The proportion of the packages will weigh more than1lb is 0.5.
b) The proportion of the packages will weigh between 0.95 and 1.05 lbs is 0.2586.
c) The probability that a randomly selected package will weigh less than 0.8 lbs is 0.0918.
d) Z=3, yes, this would be unusual to find a package of that size.

6.21 We are given Cerebral blood flow in the brains of healthy people is normally distributed with a mean of 74 and a standard deviation of 16.

a) The probability that a healthy person will have CBF reading between 60 and 80 is 0.4586.
b) The probability that a healthy person will have a CBF of above 100 is 0.0526.
c) The probability that a healthy person will be mistakenly diagnosed as at risk is 0.0170.

6.38 a) Since $np = 15*0.5=7.5$ and $nq=15*0.5=7.5$, both $np>5$ and $nq>5$, so the normal approximation is appropriate.

b) $\mu = np = 7.5$, $\sigma = \sqrt{npq} = 1.9365$, so $P(X \geq 6) = 1-P(X \leq 5) \approx 1-P(Z \leq \frac{5.5-\mu}{\sigma})$
$= 1-P(Z \leq -1.03) = 1-0.1515 = 0.8485$.

c) $P(X>6)=1-P(X \leq 6) = 1- P(Z \leq -0.52) = 1-0.3015 = 0.6985$.

d) The approximations from b) and c) are very close to the exact probabilities.

6.46 Since $p=0.001$, $q= 0.999$ and $n=50000$, and both $np>5$ and $nq>5$, so it is appropriate to use the normal approximation to the binomial probability, then $\mu = 50$ and $\sigma = \sqrt{npq} = 7.068$,

$P(X \geq 60) \approx P(Z \geq \frac{59.5-50}{7.068}) = P(Z \geq 1.34) = 1- 0.9099 = 0.0901$, therefore, the probability of there being 60 children with genetic defects is low, but now so low as to considered rare event.

6.52 a) $P(X>20) = 1-P(X \leq 20) = 1-0.991 = 0.009$

b) Using the normal approximation to the binomial distribution:

$P(X>20)=1-P(X\le 20)\approx 1-\ P(Z\le \frac{20.5-15}{2.449})$ =1-0.9878=0.0122, compart to part a), they are quite close.

c) n=25 and p=0.8 , by using Table 1 in Appendix I,

so $P(15\le X\le 20) = P(X\le 20)-P(X\le 14)=0.579-0.006 =0.573$

d) Using the normal approximation to the binomial distribution to approximate the probability, so $P(15\le X\le 20)\approx P(\frac{14.5-20}{2}\le Z\le \frac{20.5-20}{2})$ =0.5987-0.0030=0.5957, therefore, the approximation is relatively close to the actual value from part c).