Tsunamis and ocean waves

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Introduction

- Tsunami waves are generated relatively often, from various sources
- Serious tsunamis (serious to the point of loss of lives) take place on the order of once every several decades
- I think that we were all personally affected by the deadly 2004 Boxing Day Tsunami generated by the very powerful earthquake off of the Sumatra coast.
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Potential Contributions

- Predicting earthquakes: a grand challenge problem, that is not presently within reach.
- Where is it currently feasible for mathematics to contribute to the problem of tsunami danger?
- modeling of tsunami wave generation
- and propagation across oceans
- and their impact on coastlines
- Design of early warning systems (or some of its components)

  including modeling in faster than real time
- Clarification of the character of tsunami waves

  in particular those features which affect tsunami-safe engineering and architecture

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Description of the Sumatra earthquake

- major earthquake on 26 December 2004, at 7h58 local time, centered approximately 250km off the west coast of the island of Sumatra
- Initially estimated at $M_w = 8.5 \sim 9.0$, and subsequently classed at $M_w = 9.3$.
- Rupture along the Sunda Trench subduction zone, of length $1200km$.
- Slip magnitude $21m$ horizontal displacement in the southern region, $\sim 15m$ in the north.
- Vertical displacement estimates from $2 - 10m$ in different regions.

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Northeastern Indian Ocean tectonic setting

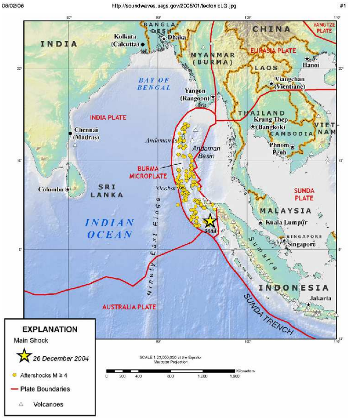

Figure: USGS website: http://soundwaves.usgs.gov/2005/01/tectonicLG.jpg

Description of the resulting tsunami

- Tsunami generated by the line source of the displacement zone

  Soon afterwards, waves impact upon the Sumatra coast
- Waves travel at $\sim 360 km/hr$ propagating east through the Andaman Basin, and in somewhat over one hour they impinge on the west coast of the Malay Peninsula, including portions of Thailand and Myanmar.
- Waves travel at $\sim 720 km/hr$ propagating west through the lower Bay of Bengal/Indian Ocean, impacting on Sri Lanka and south India in approximately 2 hours.
- The tsunami is detected globally; within 7 hours on the African Coast, 20 hours in Rio de Janeiro, 23 hours in Chile, and 29 hours in Halifax, Nova Scotia.

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The global reach of the 26 December 2004 Sumatra tsunami

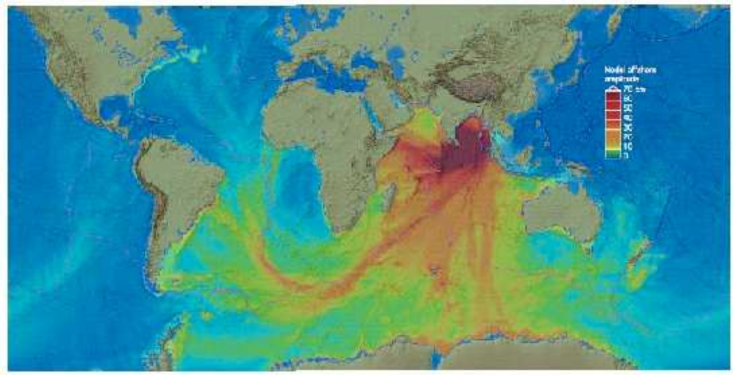

Figure: Science Express on 25 August 2005: Science 23 September 2005, Vol. 309. no. 5743, pp. 2045 - 2048 Vasily Titov, Alexander B. Rabinovich, Harold O. Mofjeld, Richard E. Thomson, Frank I. González

Bathymetry of the northeastern Indian Ocean

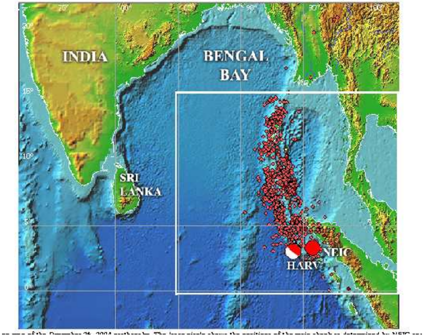

Figure: Novosibirsk Tsunami Laboratory website: http://tsun.sscc.ru/tsulab/20041226.htm

NB Depth of the Indian Ocean $\sim 3K - 4Km$.
Depth of the Andaman Basin $\sim 1Km$.

Three aspects of modeling of large ocean waves

- Generation of the disturbance
  - point sources
  - line sources
  - focusing effects
- Propagation over large distances, at high velocities
  - varying bottom depth
  - waveguide effects by mid-oceanic ridges
- Incidence on the coastal regions
  - reflection by the continental shelf
  - refraction
  - run-up amplification on shorelines
  - resonance effects

Equations of motion

- potential flow $\nabla \cdot u = 0$, $\nabla \times u = 0$, implying $u = \nabla \varphi$.

  $$\Delta \varphi = 0 \tag{1}$$

  in a three dimensional fluid region $\{h(x) < y < \eta(x,t), x \in \mathbb{R}^2\}$.
- free surface boundary conditions; $N = (-\partial_x \eta, 1)$ outward normal

  $$\partial_t \eta - N \cdot \nabla \varphi \ , \qquad \partial_t \varphi = -g\eta - \tfrac{1}{2}|\nabla \varphi|^2 \tag{2}$$
- bottom boundary conditions; $N = (\partial_x h(x), -1)$ outward normal

  $$N \cdot \nabla \varphi = 0 \tag{3}$$
- moving bottom boundary conditions; let N outward unit normal

  $$N \cdot \nabla \varphi = -\partial_t h(x,t) \qquad \text{the source} \tag{4}$$

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Linear wavespeed

- The wavespeed of long waves (Stokes 1847)

$$c = \sqrt{gh} \, , \qquad h = \text{depth} \tag{5}$$

- Free surface hydrodynamical equations, linearized

$$\Delta\Phi = 0 \, , \qquad N \cdot \nabla\Phi|_{\text{bottom}} = 0 \, ,$$

$$\partial_t \eta = \partial_y \Phi \, , \qquad \partial_t \Phi = -g\eta \, , \qquad \text{on} \quad y = -h$$

- Describe $\partial_y \Phi$ with the Dirichlet - Neumann operator

$$G : \Phi(x,0) \mapsto \Phi(x,y) \;=\; \text{the Poisson extension}$$

$$\mapsto \nabla\Phi \cdot N_\eta := (G\Phi)(x)$$

  The linear operator $G = G(h(x))$ depends upon $h(x)$

- In case h is a constant depth,

$$G = |D| \tanh(h|D|) \, , \quad \text{where} \quad D = \frac{1}{i}\partial_x$$

Phase and group velocities

- The above linear analysis gives to the dispersion relation

$$\omega(k) = \sqrt{gk|\tanh(h|k|)} \,, \qquad c_p = \frac{\omega(k)}{|k|}\frac{k}{|k|} \,, \quad c_g = \partial_k \omega(k) \tag{6}$$

- For large wavelength disturbances

$$|c_p| = lim_{k \to 0} \frac{\omega(k)}{|k|} = \sqrt{gh} = |c_g|$$

Comparison with observations

- Group and phase velocities of longest waves;

$$|c_p| = lim_{k\to 0}\frac{\omega(k)}{|k|} = \sqrt{gh} = |c_g|$$

  The acceleration of gravity is $g = 9.8m/sec^2$

- In the Bay of Bengal/Indian Ocean, where $h \sim 4Km$.

$$c_p = \sqrt{9.8 \times 4 \times 10^3 m^2/sec^2} \sim 200m/sec = 720km/hour$$

  Travel time from epicenter to the coast of Sri Lanka ($1550km$) is 2.2 hours.

- Andaman Basin, $h \sim 1Km$.

$$c_p = \sqrt{9.8 \times 10^3 m^2/sec^2} \sim 100m/sec = 360km/hour$$

  Travel time to the Malay Peninsula ($700Km$) is 1.9 hours.

Modeling of nonlinear waves

- Nonlinearity in the equations of surface wave propagation gives rise to the possibility that waves can propagate in coherent wave packet, with little loss of amplitude.
- Observers of the impact of the Sumatra tsunami reported the arrival of 6 or 7 major crests (on *e.g.* the Thai coast), with periods of 20 to 30 minutes.
- Wave amplitudes observed $80cm$ at sea, amplified to $3 - 7m$ on the coast.
- Deducing the general character of the waveform from this data

$$\begin{aligned} &\text{wavelength} \quad \ell = 180km, \\ &\text{slope} \quad \partial_x \eta \sim 2 \times 10^{-5} := \varepsilon. \end{aligned}$$

Comparing dispersion and nonlinearity

- Dimensionless scaling parameters $\alpha := a/h$ and $\beta := h/\ell$, where a is the amplitude, h the typical depth, and ℓ the characteristic wavelength.

  dispersive nonlinear regime $\alpha \sim \beta^2 << 1$

  weakly nonlinear shallow water regime $\alpha \sim \beta << 1$
- Model equations with dispersive and nonlinear character - KdV (or Boussinesq) equations.

$$\partial_t r = c_0 \partial_x r - \frac{3\alpha}{2} c_1 r \partial_x r - \frac{\beta^2}{6} c_2 \partial_x^3 r + \beta^2 f(r) \qquad (7)$$

- Linear wavespeed $c_0 = c_0(x;\omega) = \sqrt{g(\overline{h})} + \beta c_0^1 + \beta^2 c_0^2$ is variable, a function of the bathymetry $h(x)$. Similarly c_1 and c_2. The source f is an integral term taking into account reflected waves of of the topographical features of the ocean basin.

Determing scaling regime

- Indian Ocean/Bay of Bengal: depth $h \sim 4Km$, amplitude $a \sim 1m$, hence $\alpha = a/h \sim 2.5 \times 10^{-4}$.

  wavelength $180Km$, hence $\beta = h/\ell \sim 4Km/180Km \sim 2 \times 10^{-2}$

  $$\beta^2 \sim \alpha$$

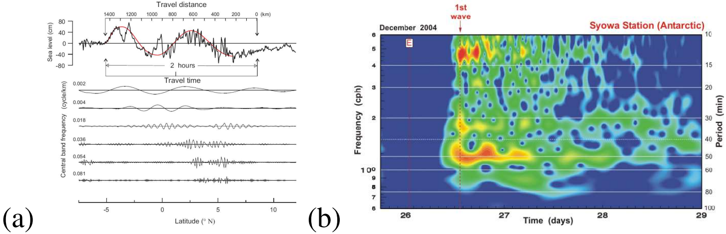
- Andaman Basin: depth $h \sim 1Km$ amplitude $a \sim 1m$, hence $\alpha = a/h \sim 10^{-3}$

  wavelength $180Km$, hence $\beta = h/\ell \sim 6 \times 10^{-3}$

  $$\beta^2 << \alpha << \beta$$

- This analysis indicates that wave propagation on these scales in the Indian Ocean is of nonlinear dispersive character, while in the Andaman Basin it is more ambiguous.

Dispersion indicated in satellite telemetry



(a) (b)

Figure: (a) from E. Kulikov, Shirshov Institute of Oceanography, Moscow and Institute of Ocean Sciences, Sydney, BC: ‘Dispersion of the Sumatra tsunami waves in the Indian Ocean detected by satellite altimetry’, Fisheries and Oceans Canada, Science - Pacific Region website (2005)
(b) From the Tsunamis and tsunami research website of the Institute of Ocean Sciences, Sidney BC.
This time/frequency plot of the Sumatra tsunami supports the wavepacket hypothesis.

Complexity of the near-shore wave dynamics

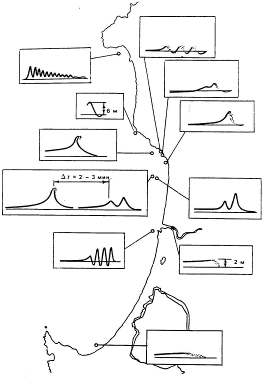


Figure: E. Pelinovsky: *Dynamics of tsunami waves* (1996)[Russian]
Nihonkai - Chubu earthquake tsunami on the north Akita coast.
N. Shuto, Coastal Eng. Japan (1985), vol. 28 pp. 255 - 264

Sri Lanka images

Figure: Shoreline of Kalutara, Sri Lanka on 26 December 2004. Photo: DigitalGlobe

Conclusions

- Further study is crucial, and mathematical theory and scientific computations can play a major rôle
- The generation of (major) tsunamis is a key (see A. Lerner-Lam, in this AAAS Symposium).
- There are methods which predict tsunami propagation in the open ocean. Task - to verify their accuracy, improve their speed.

  Specific questions: propagation over random bottom topography, reflection at the continental shelf.
- Much needs to be done on run-up and other aspects of impact when a major tsunami is incident on a coastline.

  Edge waves, and phenomena associated with other coast geometries which focus and otherwise amplify an incident large wave.

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