Walter Cras

The mathematic

Reno War

the

Hokusai
Magnitude 9.0 - SUMATRA-ANDAMAN ISLANDS
EARTHQUAKE
OFF THE WEST COAST OF NORTHERN SUMATRA
2004 December 26 00:58:53 UTC

Magnitude 9.0
Date-Time
Sunday, December 26, 2004 at 00:58:53 (UTC)
= Coordinated Universal Time
= local time at epicenter
= Time of Earthquake in other Time Zones

Location
3.316°N, 95.854°E

Depth
30 km (18.6 miles) set by location program

Region
OFF THE WEST COAST OF NORTHERN SUMATRA

Distances
250 km (155 miles) SSE of Banda Aceh, Sumatra, Indonesia
310 km (195 miles) W of Medan, Sumatra, Indonesia
1260 km (780 miles) SSW of BANGKOK, Thailand
1605 km (990 miles) NW of JAKARTA, Java, Indonesia

Location Uncertainty
horizontal +/- 5.6 km (3.5 miles); depth fixed by location program

Parameters
Nst=276, Nph=276, Dmin=654.9 km, Rmss=1.04 sec, Gp=29°
M-type=teleseismic moment magnitude (Mw), Version=U

Source
USGS NEIC (WDCS-D)

Event ID
usslav

Felt Reports: At least 79,900 people were killed by the earthquake and tsunami in Indonesia. Tsunamis killed at least 41,000 people in Sri Lanka, 10,000 in India, 4,000 in Thailand, 120 in Somalia, 90 in Myanmar, 66 in Malaysia, 46 in Maldives, 10 in Tanzania, 2 in Bangladesh, 1 in Seychelles and 1 in Kenya. Tsunamis caused damage in Madagascar and Mauritius and also occurred on Cocos Island and Reunion. The tsunami crossed into the Pacific Ocean and was recorded in New Zealand and along the west coast of South and North America. The earthquake was felt (VIII) at Banda Aceh and (V) at Medan, Sumatra and (II-IV) in parts of Bangladesh, India, Malaysia, Maldives, Myanmar, Singapore, Sri Lanka and Thailand. A mud volcano near Batangas, Andaman Islands began erupting on December 28. This is the fourth largest earthquake in the world since 1900 and is the largest since the 1964 Prince William Sound, Alaska earthquake. (last updated 12/30/04)

The devastating megathrust earthquake of December 26th, 2004 occurred on the interface of the India and Burma plates and was caused by the release of stresses that develop as the India plate subducts beneath the overriding Burma plate. The India plate begins its descent into the mantle at the Sunda trench which lies to the west of the earthquake's epicenter. The trench is the surface expression of the plate interface.
Northeast Indian Ocean Region
Tectonic Setting

EXPLANATION
Main Shock
26 December 2004
© Michael D. 4
Plate Boundaries
Volcanoes

- depth: 20 km
  horizontal displacement
  20 m at this depth
  10 m on sea bed
  2 m on land
- vertical displacement
  major rupture
  400 km length, Simul Falt
  1500 km length
The location map of the December 26, 2004 earthquake. The large circle shows the positions of the main shock as determined by NEIS along with the first day aftershocks that roughly outline the earthquake source area. The main plate boundaries are shown in green. White rectangle outlines the area shown in the section "Source area".
§1. Equations of motion

\[ \Delta \varphi = 0 \]

(a) \[ \nabla \cdot \mathbf{u} = 0 \]
\[ \nabla \times \mathbf{u} = 0 \quad \mathbf{u} = \nabla \varphi \]

conditions for potential flow

(b) \[ \Delta \varphi = 0 \]

bottom boundary condition
\[ N \cdot \nabla \varphi = \Theta - \gamma b(x, t) \quad \gamma = b(x, t) \]

Top boundary condition, \( \mathbf{n} \)
\[ \gamma = \gamma(x, t) \]

(3) \[ \hat{\mathbf{n}} = (-\partial_1 \varphi, -\partial_2 \varphi, 1) \] space-time normal
\[ \hat{\mathbf{t}} = (1, \partial_1 \varphi, \partial_2 \varphi) \] tangent vector to fluid particle path
\[ \hat{\mathbf{n}} \cdot \hat{\mathbf{t}} = 0 \]
That is, we obtain the kinematic condition
\[ \nabla \cdot \mathbf{u} - \mathbf{u} \cdot \nabla \mathbf{u} + \gamma \mathbf{u} = 0. \]

A second boundary condition on the free surface
\[ \nabla \cdot \mathbf{u} + g \gamma + \frac{1}{2} \rho \frac{d}{dt} \mathbf{u}^2 = 0 \]
the dynamic or Bernoulli condition.

Compare (51) with the Euler equations
\[ \nabla \cdot \mathbf{u} + (\mathbf{u} \cdot \nabla \mathbf{u}) = -\nabla p - g \gamma (\theta) \]
and integrate once for a condition on \( \varphi, \Delta \nabla \varphi \mathbf{u} \).

Questions: how can an earthquake create waves of large mass, momentum and energy, which travel at great velocity in a coherent form?
What happened in the Dec 26, 2004 earthquake and the resulting tsunami from north east Sumatra?

- Earthquake 8.95° local time
  - 100 km offshore north Sumatra
- Soon afterwards, impact of tsunami on Sumatra coast
  - Baya Area
- Tsunami waves travel at ~360 km/hr east through the Andaman Basin, arriving in just over one hour on the Raylay peninsula coast.
- Tsunami waves travel at ~720 km/hr west through the Bay of Bengal to impact the west coast of Sri Lanka and the west coast of southern India in approximately 2h 30min (10° local time).

Basic explanations for:

1. wave speed, and thus the travel time
2. wave form of the coherent wave in the form of a wave packet which forms the tsunami phenomenon.
§2 Linearized Equations

The wave speed of free surface water waves is

\[ c = \sqrt{\frac{g}{\gamma}} \]

[Stokes, 1847]

Tasks: describe this

* The hydrodynamic equations, linearized about \( \eta(x,t) = 0 \)

\[ \Phi(x,t) = 0 \quad \text{for} \quad -b < \gamma < 0, \]

\[ N \cdot \Phi = 0 \quad \text{for} \quad \gamma = -b \]

* Linearized free surface conditions

\[ \partial_t \eta = \partial_t \Phi \quad \text{on the linearized free surface} \]

\[ \partial_t \Phi = -g \gamma \quad \gamma = 0 \]

* Describe \( \partial_t \Phi \) using the Dirichlet-Neumann operator

\[ \Phi(x,t) \quad \text{boundary data} \]

\[ \rightarrow \Phi(x,\gamma) \quad \text{such that} \]

\[ \Delta \Phi = 0 \]

\[ N \cdot \Phi = 0 \quad \gamma = -b \]

Poisson extension

\[ \rightarrow \partial_t \Phi(x,\gamma) = G \Phi(x,\gamma) \]

normal derivative at free surface

The operator \( G = \partial_t \Phi(x,\gamma) \), linear in \( \Phi \) coefficients in \( b \).
Re-writing the linearised equation

\[ \frac{\partial^2 \zeta}{\partial t^2} = -\gamma \frac{\partial^2 \zeta}{\partial x^2} \]

separate variables \( \zeta(x,t) = e^{i\omega t} \phi(x) \), then

\( \phi(x) \) satisfies problem for the Dirichlet-Neumann operator.

Case of a flat bottom \( b(x) = -h \).

\[ \begin{cases} \phi(0) = e^{ikx} \\ \phi'(0) = 0 \end{cases} \]

- Data
  \[ \Phi(x,0) = e^{ikx} \]

- Poisson extension
  \[ \Phi(x,y) = (a e^{iky} + b e^{-iky}) e^{ikx} \]

- Bottom boundary condition
  \[ \Phi(x,y) = \frac{\cosh \left( kh \left( y + h \right) \right) e^{ikx}}{\cosh \left( kh \right)} \]

- Gradient on the free surface
  \[ G e^{ikx} = \frac{kh \sinh \left( kh \right)}{\cosh \left( kh \right)} e^{ikx} \]
  \[ = \frac{kh \tanh \left( kh \right)}{\cosh \left( kh \right)} e^{ikx} \]
For general data, a linear superposition
\[ \tilde{S}(\xi) = \int \frac{1}{\sqrt{2\pi}} e^{i\xi \cdot x} \frac{1}{2} \left( \begin{array}{cc} \cosh x & \sinh x \\ \sinh x & \cosh x \end{array} \right) \tilde{F}(\xi) \, d\xi \]

Using the notation of Fourier multipliers
\[ (G \tilde{S})_{\xi} = \sqrt{D} \tanh (\frac{\xi}{D}) \tilde{S}(\xi) \]
where
\[ D = \frac{1}{2} \Theta x \]

* Back to the spectral problem

One has
\[ \omega^2 c^2 = g \tanh (\frac{\xi}{D}) \]

Solution of the wave equation (flat bottom)
\[ \partial_x^2 \eta = -g \, C(0) \eta \]
given by
\[ \eta(x,t) = \frac{1}{\sqrt{2\pi}} \int e^{i\xi \cdot x} \left[ \cos (\omega_c(\xi) t) \tilde{\eta}_0(\xi) + \sin (\omega_c(\xi) t) \frac{\tilde{\eta}_0(\xi)}{\omega_c(\xi)} \right] \, d\xi \]
with of course
\[ \omega_c(\xi) = \sqrt{g \tanh (\frac{\xi}{D})} \]
Analysis of (11) by the method of stationary phase

Phase velocity
\[ c_p = c_p(k) = \frac{\omega(k)}{k} \left( \frac{k}{1k} \right) \]

Group velocity
\[ c_g(k) = \delta \omega(k) \]

**Significance:**

Water waves case

\[ \omega(k) = \sqrt{gkh \tanh(kh)} \]

Phase velocity
\[ \frac{\omega(k)}{k} \sim \left\{ \begin{array}{ll} \frac{\sqrt{3}}{2kh} & k \rightarrow 0 \\ \frac{\sqrt{3}}{2} & k \rightarrow \infty \end{array} \right. \]

Group velocity
\[ \frac{\delta \omega(k)}{k} \sim \left\{ \begin{array}{ll} \frac{\sqrt{3}}{2kh} & k \rightarrow 0 \\ \frac{\sqrt{3}}{2} & k \rightarrow \infty \end{array} \right. \]

Conclude: wave speed for long waves is \( \sqrt{gh} \).

Singularities (short waves) do not propagate.
Sample scratch calculations

(i) Sumatra coast \rightarrow South India 2,500 km
Bay of Bengal \sim 3K - 4K m depth

\[ g = 9.8 \text{ m/sec}^2 \]

\[ c = \sqrt{gh} \sim \sqrt{10 \text{ m/sec}^2 \times 4 \times 10^3 \text{m}} \]

\[ = 2 \times 10^2 \text{ m/sec} \]

\[ = 220 \text{ km/hr} \]

To travel 2,500 km it would take approximately 3.5 hrs

(ii) Sumatra coast \rightarrow Malay peninsular 500 km
Andaman Basin \sim 1K m depth

\[ c = \sqrt{gh} \sim \sqrt{10 \text{ m/sec}^2 \times 10^3 \text{m}} \]

\[ = 10^2 \text{ m/sec} \]

\[ = 360 \text{ km/hr} \]

Travel time for 500 km
1.8 hrs

These somewhat overestimate the time of travel a Dec 26.
Hypothetical situation:

Situation: If the source of the tsunami wave is in the Gulf of Thailand.

- Raylay peninsula to coast of central Thailand, 500 km
- Gulf of Thailand depth ~ 200 m

\[ \sqrt{gh} = \sqrt{10 \text{ m/s}^2 \times 200 \text{ m}} \]

~ 45 m/sec

= 160 km/hr

more than 4 hrs travel time
Arrival Time of First Wave (hours) - 2004.12.26 Indonesian Tsunami

T (SECONDS) : 30 to 360.30

Source: Mw 9.0 (4.5°N, 95.7°E, 20km, 206x150km, 90° rake, 13° dip, 500° strike, 5m depth)

(7.355°N, 141.3°E, 20km, 670x150km, 90° rake, 13° dip, 345° strike, 5m depth) x (11.8°N, 43.7°E, 20km, 300x150km, 90° rake, 13° dip, 365° strike, 5m depth)

Facility for the Analysis and Comparison of Tsunami Simulations (FACTS)
* method of stationary phase

phase velocity:

\[ e^{i[k \cdot x - \omega(k) t]} = e^{i \left[ k \cdot \left( \frac{x}{t} - \frac{\omega(k)}{\omega(k)} \cdot \frac{k}{|k|} \right) \right]} \]

the phase is

\[ i t \left[ k \cdot \left( \frac{x}{t} - \frac{\omega(k)}{\omega(k)} \cdot \frac{k}{|k|} \right) \right] \]

group velocity:

\[ u(x, t) = \int e^{i[k \cdot x - \omega(k) t]} \hat{\omega}(k) \text{dk} \]

suppose \((x, t)\) is such that for all \(k \in \text{supp} \hat{\omega}\)

\[ |x - \omega(k) t| > R \]

then

\[ e^{i[k \cdot x - \omega(k) t]} = \frac{x - \omega(k) t}{|x - \omega(k) t| + i} \frac{1}{2} \partial_x (e^{i k \cdot x - t}) \]

in the integral

\[ u(x, t) = \int e^{i[k \cdot x - \omega(k) t]} \left( \frac{1}{2} \partial_x \left( \frac{x - \omega(k) t}{|x - \omega(k) t| + i} \right) \hat{\omega} \right) \text{dk} \]

\[ \leq O \left( \frac{1}{R^N} \right) \quad \forall N \]
For the present purpose we define a wave packet as having Fourier transform localized near a given wave number, say $k_0$.

$$\hat{\psi}(k) \approx e^{ikx} \psi(x), \quad \hat{k} \in C^0$$

The previous calculation shows that for wave packets

$$t \to 0$$

In this regime the estimate holds

$$|\psi(x,t)| < \frac{C^2}{R^2}, \quad x \in C^0$$

Question: linear waves exhibit dispersion, resulting in temporal decay. The nonlinearity of the water wave problem accounts for the propagation of coherent wave packets at velocities $\sim c_g(k_0)$ over long distances.
§3. Coherent waves.

Regarding the 26 Dec 2004 Sumatra earthquake and tsunami, there is almost no published data.

Most of my statements come from listening to firsthand reports in the news, photos, and from P. Krishna (IIT Bombay).

Certain facts:

- 6 or 7 waves observed (e.g.,)
  - in one case (That's coast)

- temporal period: \( T \approx 10-15 \text{ min} \)

- wave amplitude:
  - 1m - 2m at sea
  - 3m - 4m on shore

Q: From this, deduce one aspect of tsunami.
Spatial period (or linear slope): 

Time period: \( T = 15 \text{ min} \)

Phase velocity: 

\[ c_p = \sqrt{gK \tanh(kh)} / k \]

Deduce wave number: 

\[ k = \frac{2\pi}{x} \]

\[ x = \frac{2\pi}{k} = c_p T \]

\[ = \sqrt{gK \tanh(kh)} \frac{T}{k} \]

That is

\[ (2\pi)^2 = gK \tanh(kh) T^2 \]

Since \( K \) will be small:

\[ (2\pi)^2 \approx gK K^2 T^2 \]

\[ = 10 \frac{\text{m}}{\text{sec}} \times (4 \times 10^3 \text{ m}) \times (900 \text{ sec}^2) K^2 \]

\[ 15 \text{ min} = 900 \text{ sec} \]

\[ K = \frac{2\pi}{18} \times 10^4 \frac{1}{\text{m}} \approx 4 \times 10^{-5} \frac{1}{\text{m}} \]

Corresponding to spatial period:

\[ x = \frac{2\pi}{K} = 180 \text{ km} \]

Slope:

\[ \phi_x (x,t) \sim \frac{2}{x} \frac{2}{18 \times 10^4 \text{ m}} \sim 2 \times 10^{-5} \approx 2. \]