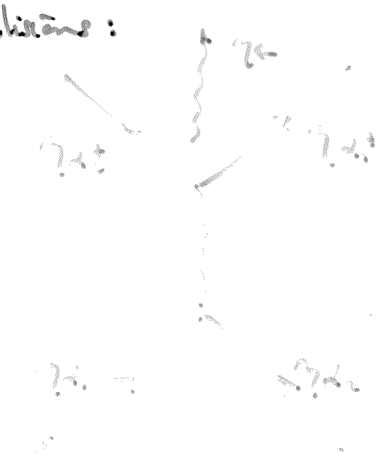
7)

* Do solitary waves behave like particles?

Schematic of collision dynamics:

* head-on collisions:

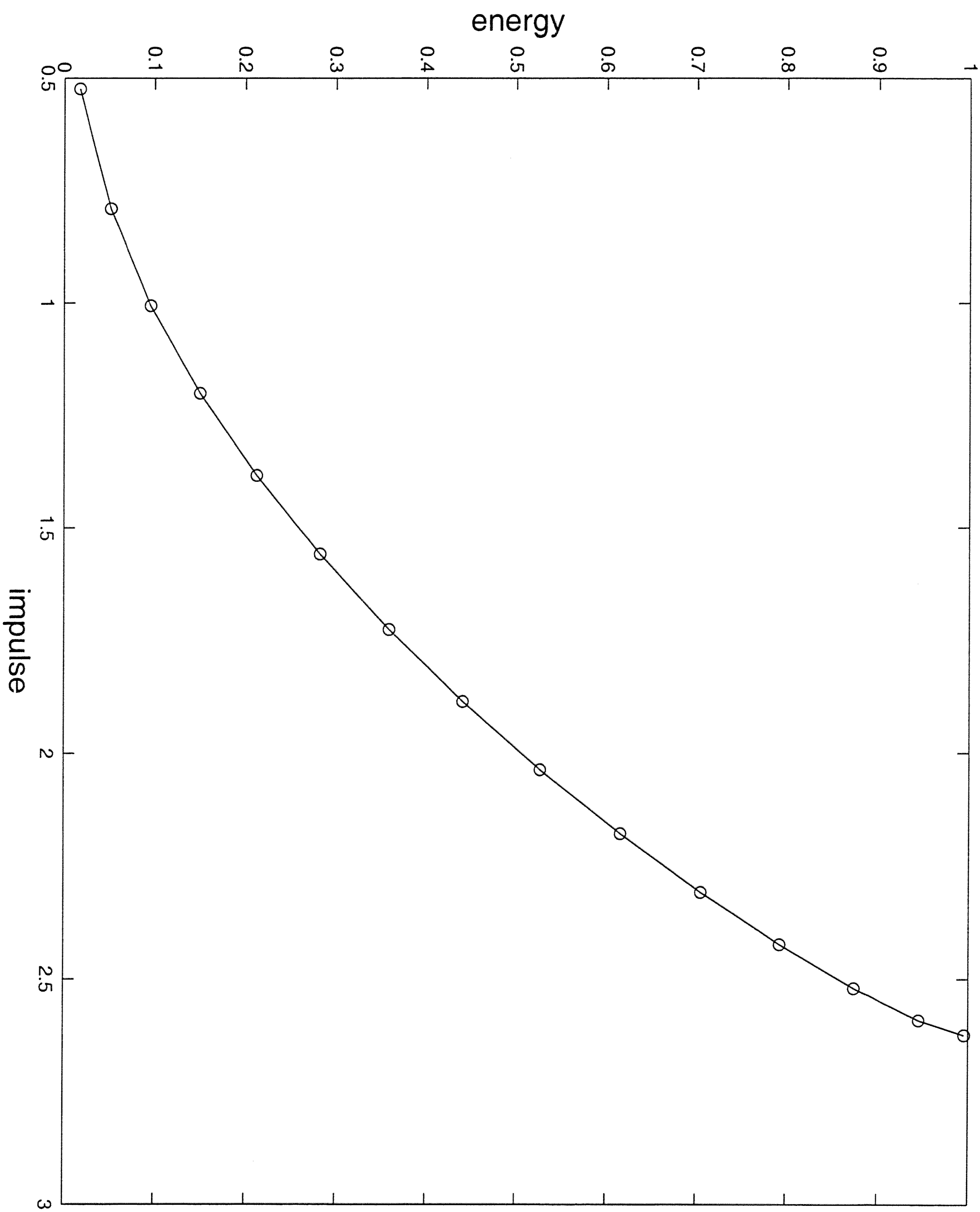


Solitons behave like attractive particles, with small loss to $\eta_R(x,t)$ from a collision.

* overtaking collisions:

Solitons behave like repelling particles, with even smaller loss to $\eta_R(x,t)$ due to collision.

Q: relationship between α_1, α_2 and α_1^+, α_2^+?



Stokes solitary wave.

10)

Recall the conservation laws:

- solitary waves $\eta_{s_\alpha}(x - c_\alpha t)$ one parameter family
- mass $m_\alpha = \int \eta_{s_\alpha}(x)\, dx$
- momentum $\mu_\alpha = \int \eta_{s_\alpha} \partial_x \xi_{s_\alpha}\, dx$
- energy $e_\alpha = H_{\text{Zakharov}}(\eta_{s_\alpha}, \xi_{s_\alpha})$

<u>Definition</u> an elastic collision is one in which

$$m_{\alpha_1} + m_{\alpha_2} = m_{\alpha_1^+} + m_{\alpha_2^+}$$
$$\mu_{\alpha_1} + \mu_{\alpha_2} = \mu_{\alpha_1^+} + \mu_{\alpha_2^+}$$
$$e_{\alpha_1} + e_{\alpha_2} = e_{\alpha_1^+} + e_{\alpha_2^+}$$

<u>Proposition</u> In an ellastic collision

(i) $\alpha_1^+ = \alpha_1$ (or α_2)

(ii) $\alpha_2^+ = \alpha_2$ (or α_1)

(iii) the residual $\eta_R(x,t) = 0$

<u>proof</u> obvious. However it depends upon some details of $(m(\alpha), \mu(\alpha), e(\alpha))$ of the family of Stokes solitary waves.

Q: Is $e(\mu)$ a convex function of μ?

11)

<u>Definition</u> An inelastic collision is one in which there is a discrepancy

$$m_T^+ := m(\alpha_1^+) + m(\alpha_2^+) = m_T - \Delta m$$

$$\mu_T^+ := \mu(\alpha_1^+) + \mu(\alpha_2^+) = \mu_T - \Delta \mu$$

$$e_T^+ := e(\alpha_1^+) + e(\alpha_2^+) = e_T - \Delta e$$

The quantities Δm, $\Delta \mu$ and Δe correspond to the mass, momentum and energy transferred to the residual $\eta_R(x,t)$.

It turns out that Δm, $\Delta \mu$, Δe cannot be arbitrary.

<u>Proposition 2.</u> Let $\alpha_1^+ = \alpha_1 - \Delta \alpha_1$ and $\alpha_2^+ = \alpha_2 - \Delta \alpha_2$.

Then

$$\begin{pmatrix} \Delta m \\ \Delta \mu \\ \Delta e \end{pmatrix} = \begin{pmatrix} m'(\alpha_1) & m'(\alpha_2) \\ \mu'(\alpha_1) & \mu'(\alpha_2) \\ e'(\alpha_1) & e'(\alpha_2) \end{pmatrix} \begin{pmatrix} \Delta \alpha_1 \\ \Delta \alpha_2 \end{pmatrix}.$$

illustration: equal amplitude counterpropagating collision, in C.M. frame

$$m(\alpha_1) = m(\alpha_2) \qquad e(\alpha_1) = e(\alpha_2) \qquad \mu_T = 0$$

By symmetry

$$\Delta \mu = \mu'(\alpha_1)(\Delta \alpha_1 - \Delta \alpha_2) = 0$$

Therefore

$$\Delta m = 2 m'(\alpha_1) \, \Delta \alpha$$

$$\Delta e = 2 e'(\alpha_1) \, \Delta \alpha$$

(2)

Theorem 3 In this case the residue $\eta_R(x,t)$ satisfies

$$\Delta e(R) = \frac{\partial e(\alpha,)}{\partial m} \Delta m(R)$$

where $\Delta m(R) = \int \eta_R \, dx$ and $\Delta e(R) = H_{Zakharov}(\eta_R, \xi_R)$

From this statement we can prove a smallness estimate on the residue

$$\left(\| \xi_R \|^2_{H^{1/2}} + \| \eta_R \|^2_{L^2} \right)^{1/2} \leq C_1 \varepsilon^4$$

This is to be compared with the known bounds on the KdV limit of the water wave equations

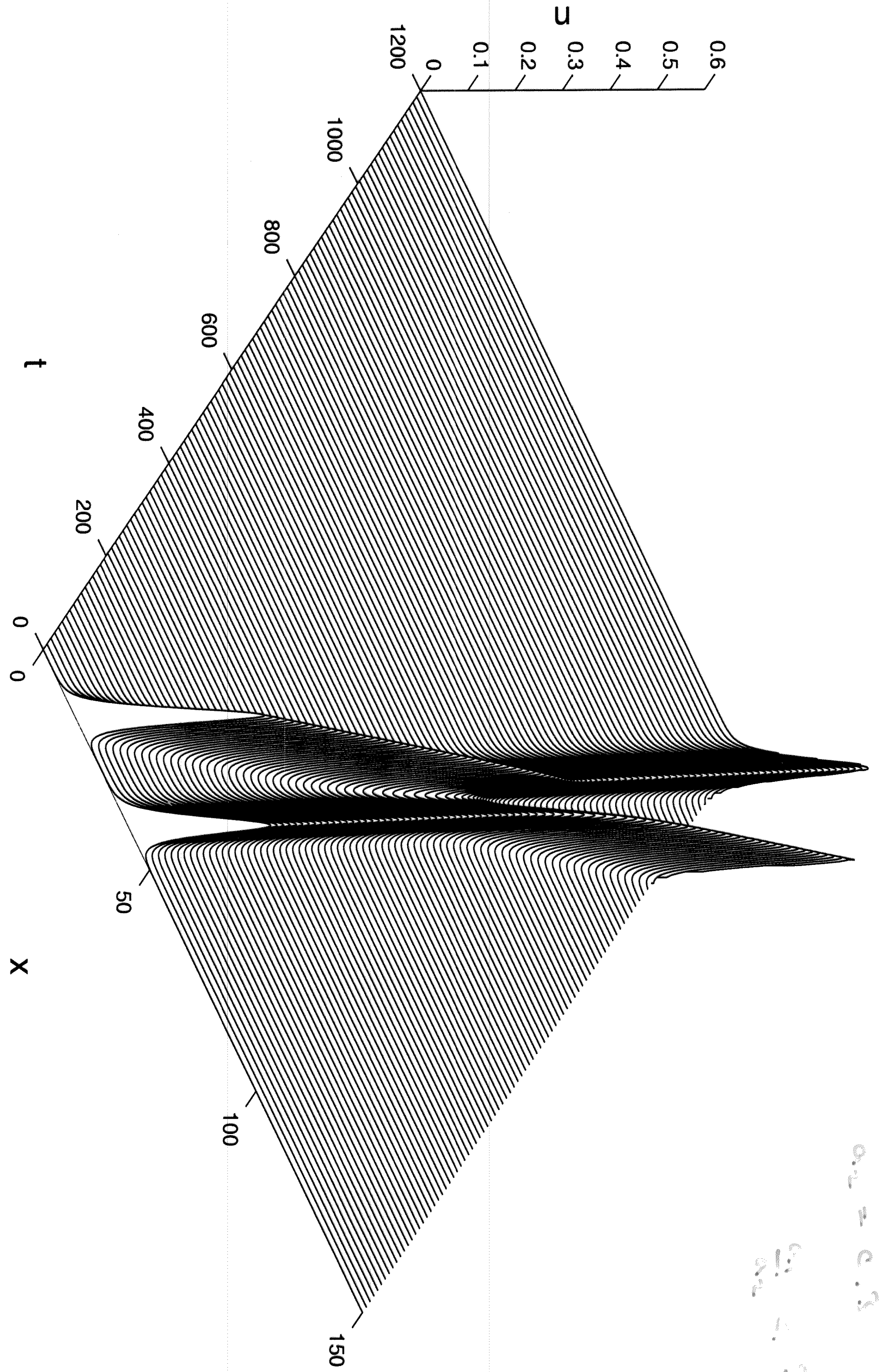
$$\| \eta(x,t) - \eta_1(x,t) \|_{H^s} \leq C_2 \varepsilon^{5/2}$$

Craig (1985)
Schneider & Wayne (2000)

where

$$\eta_1(x,t) = \varepsilon^2 \left(A_+(\varepsilon(x - c_+t)) + A_-(\varepsilon(x - c_-t)) \right)$$

made up of two KdV solutions.



K.dV solitons

$a_1 = 0.4$

$a_2 = 0.3$

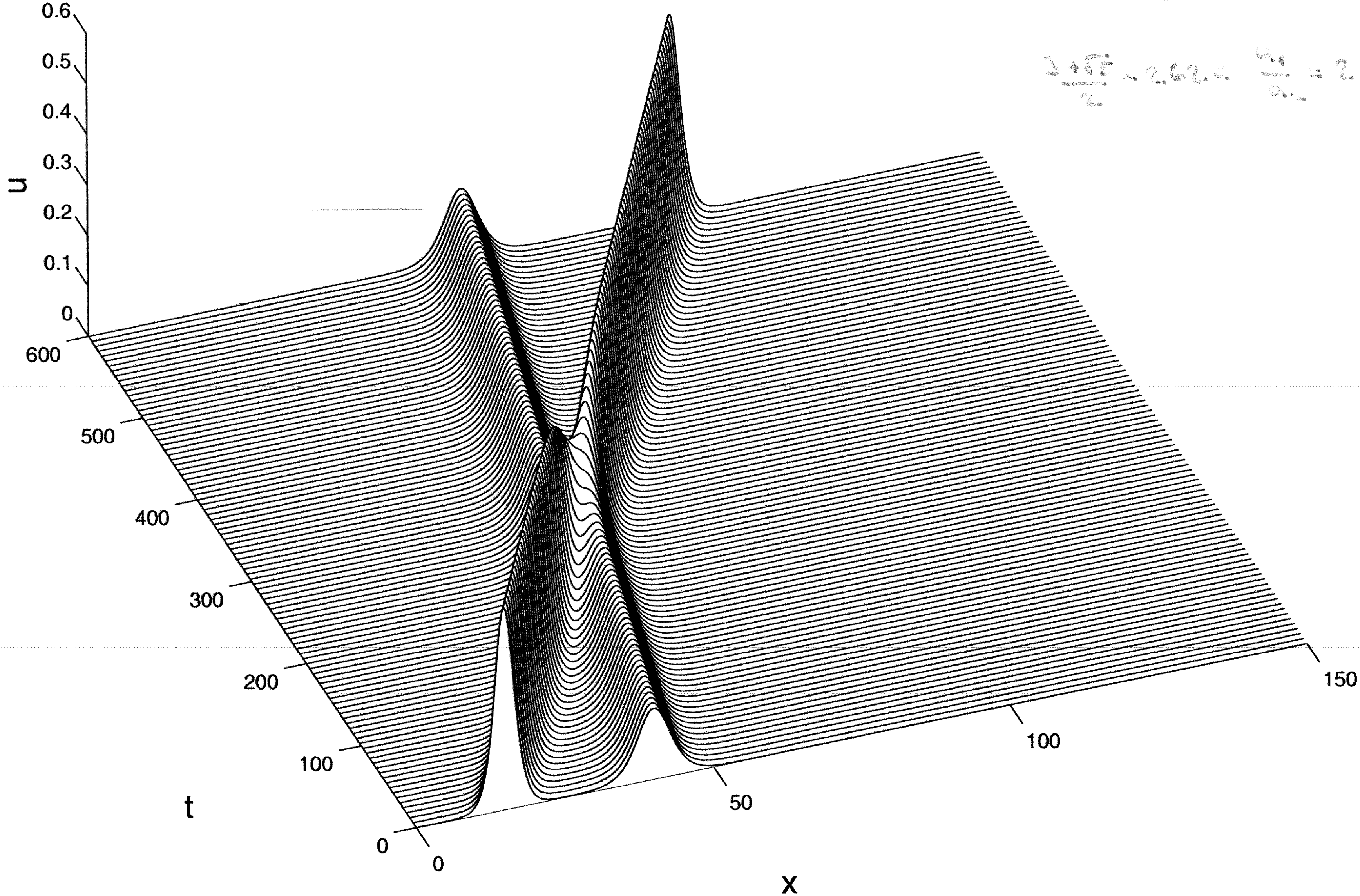
$\frac{a_1}{a_2} < \frac{3+\sqrt{5}}{2}$

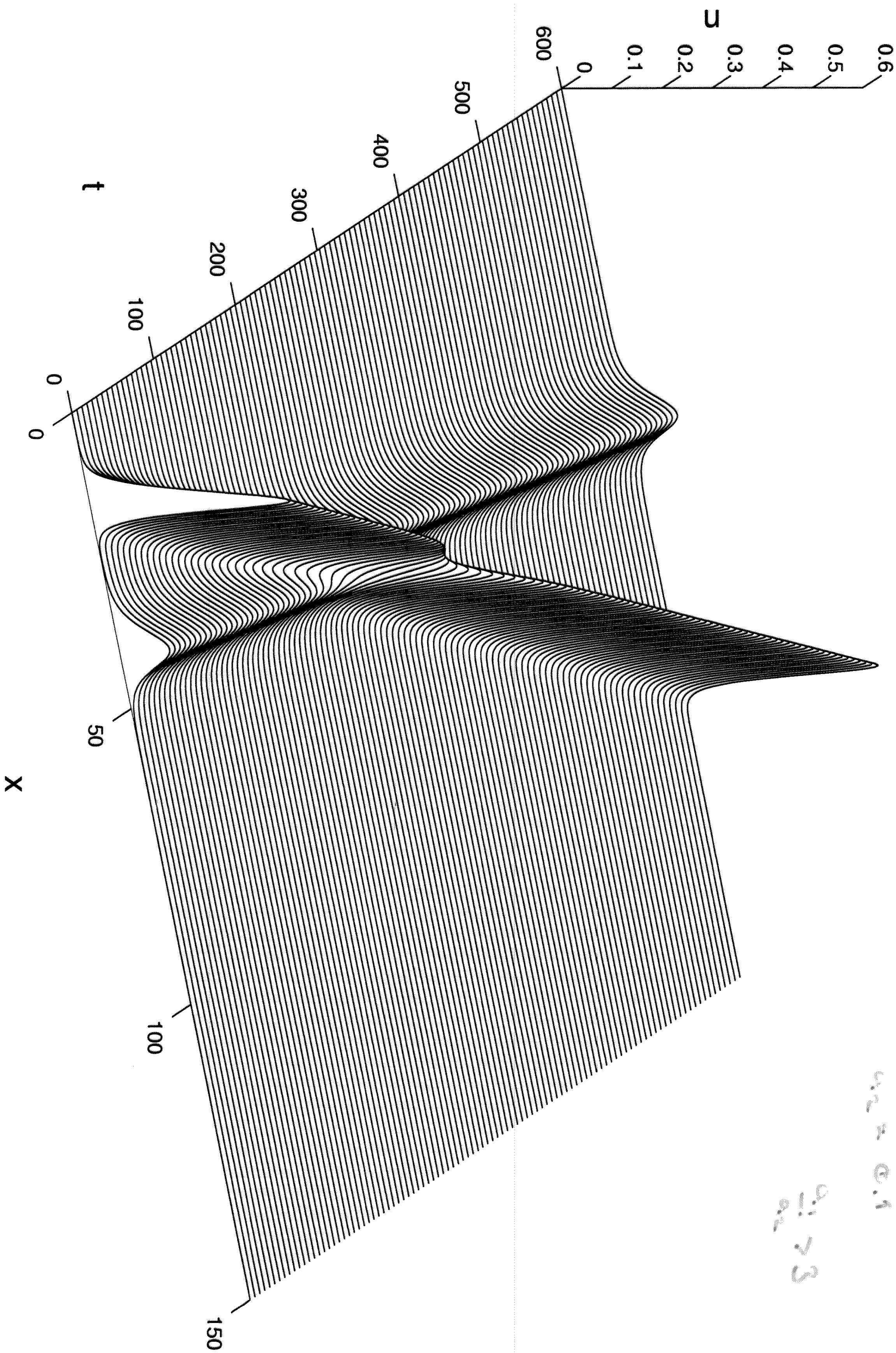
KdV solitons

$a_1 = 0.4$

$a_2 = 0.14$

$\frac{3+\sqrt{5}}{2} \approx 2.62 < \frac{a_1}{a_2} = 2.86 < 3$





KdV solitons
$c_1 = 0.4$
$c_2 = 0.1$

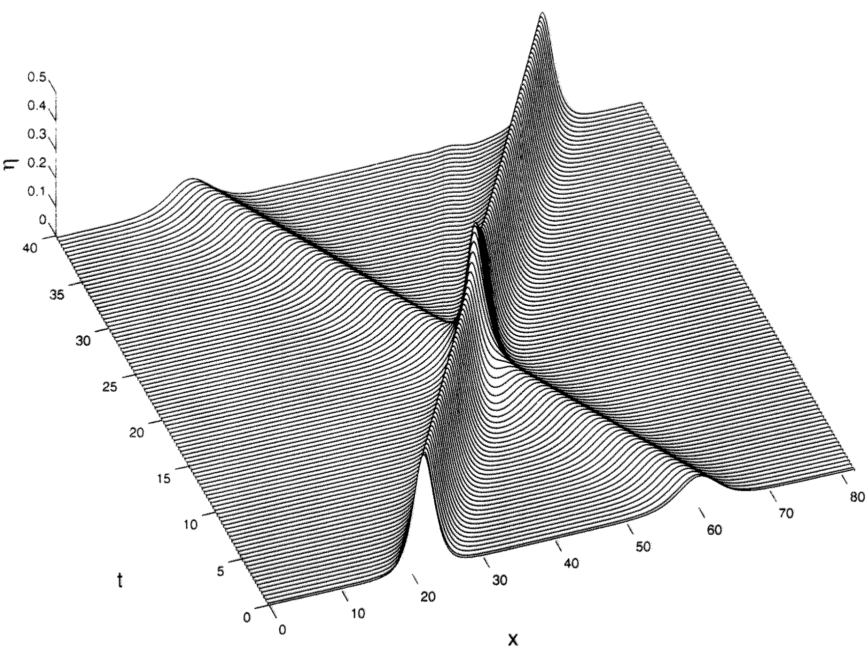
Numerical results

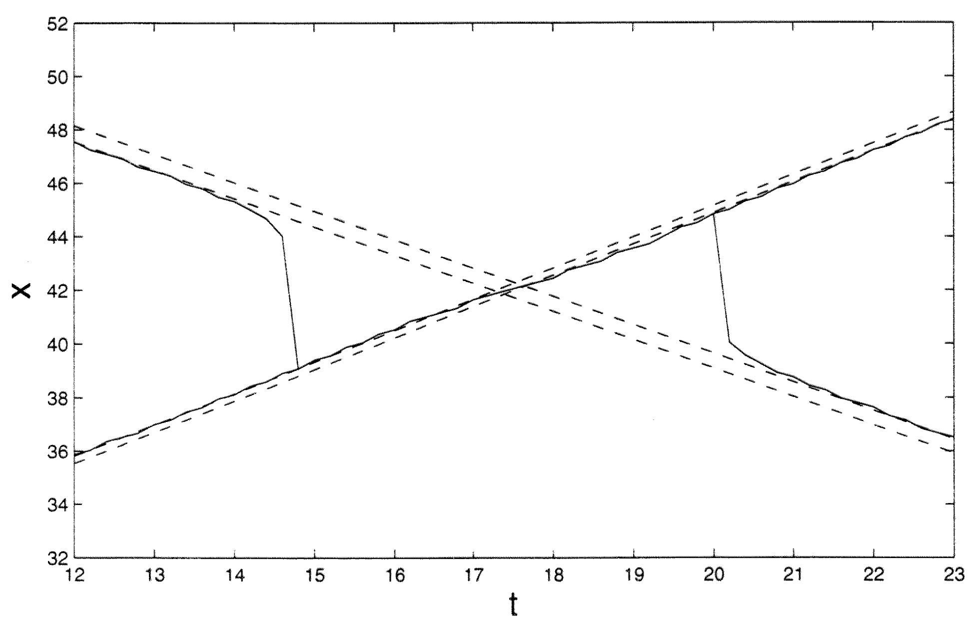
- Asymmetric head-on collisions

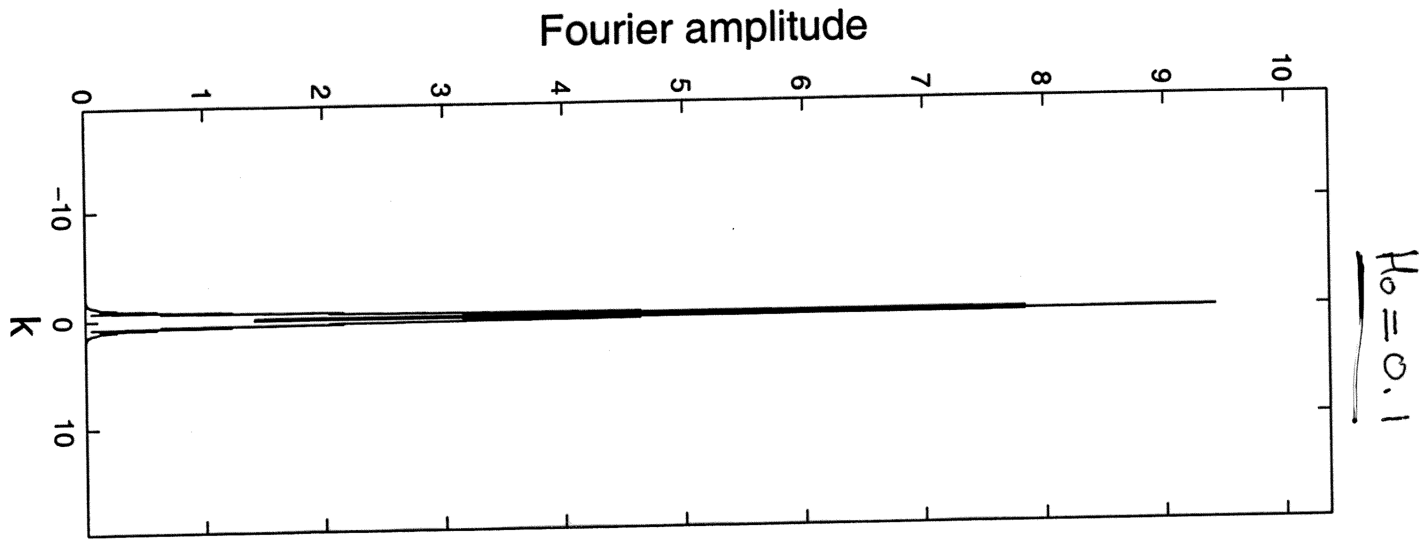
Initial amplitudes

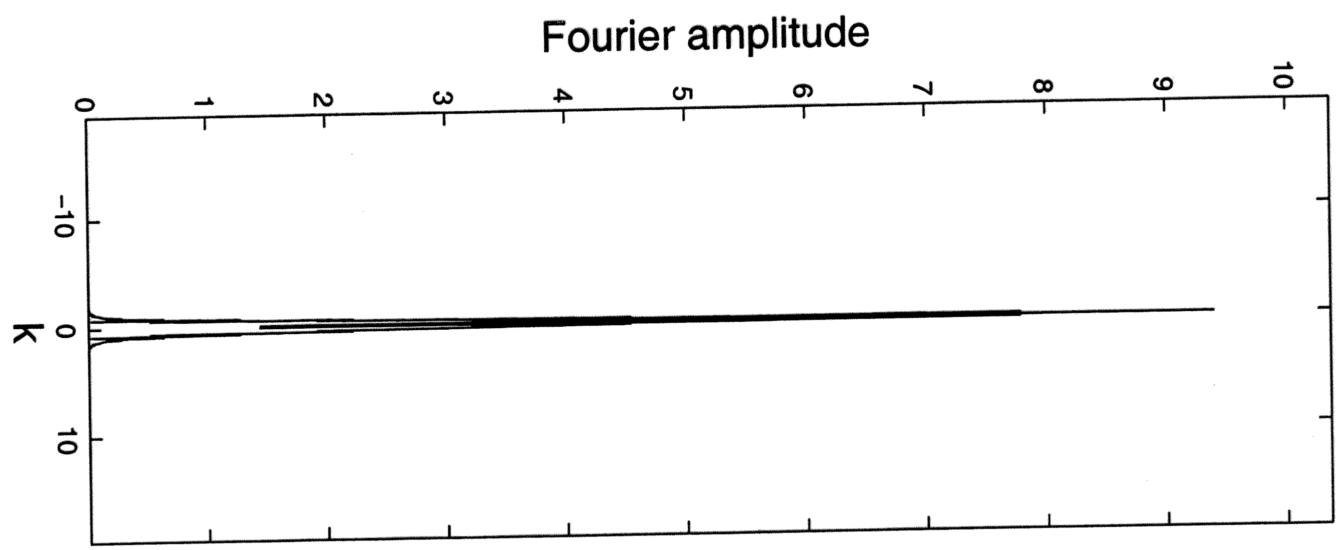
$a_1 = 0.4h$

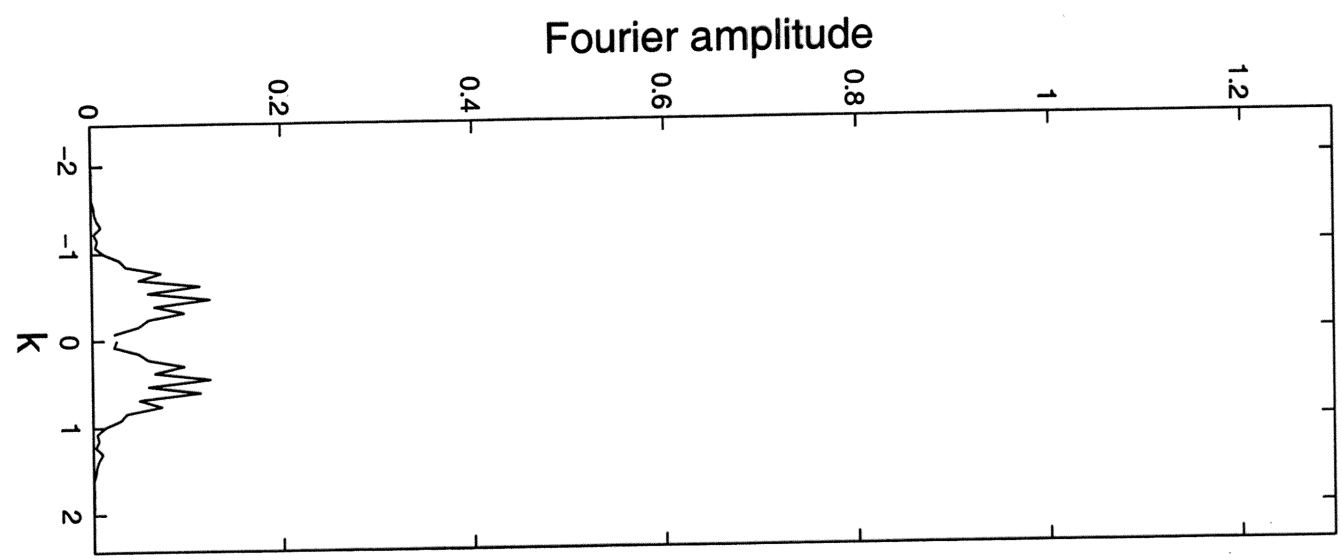
$a_2 = 0.1h$

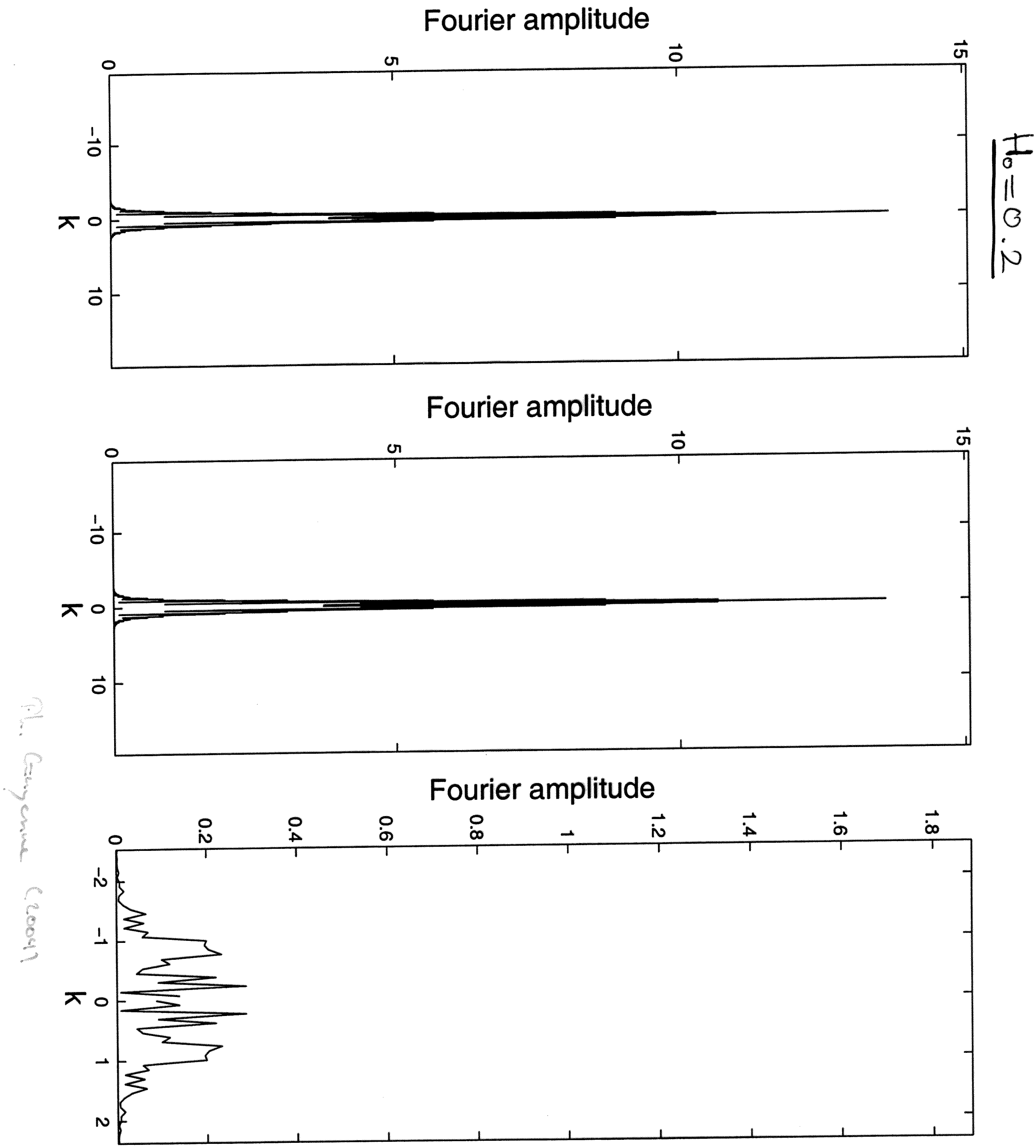




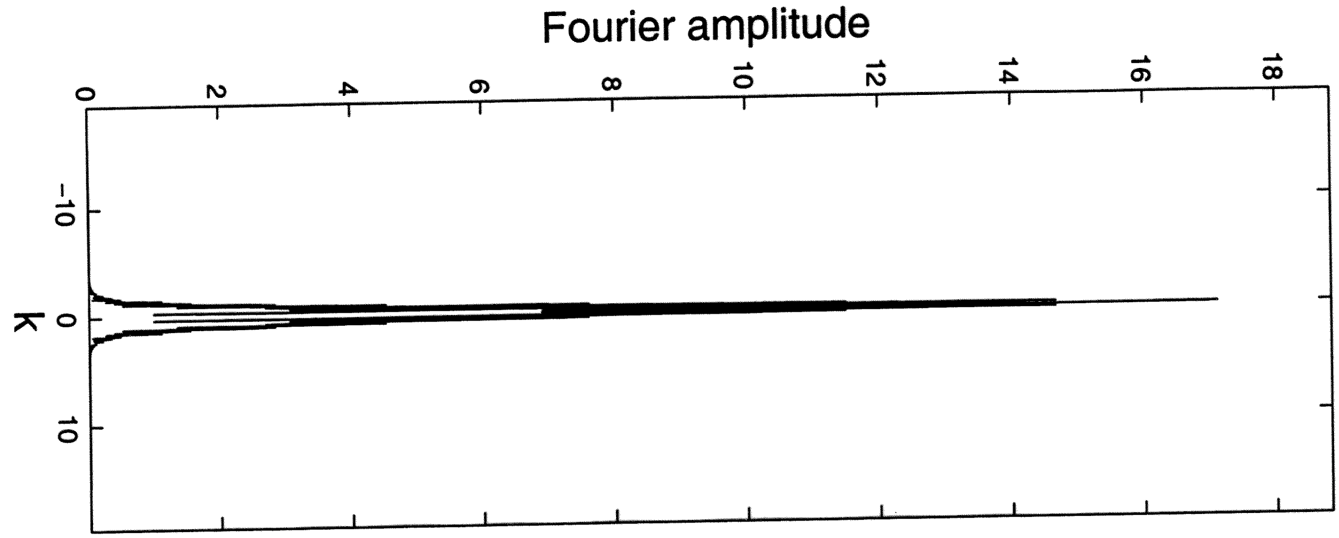

Fourier amplitude
0
1
2
3
4
5
6
7
8
9
10
-10
0
10
k
H0 = 0.1

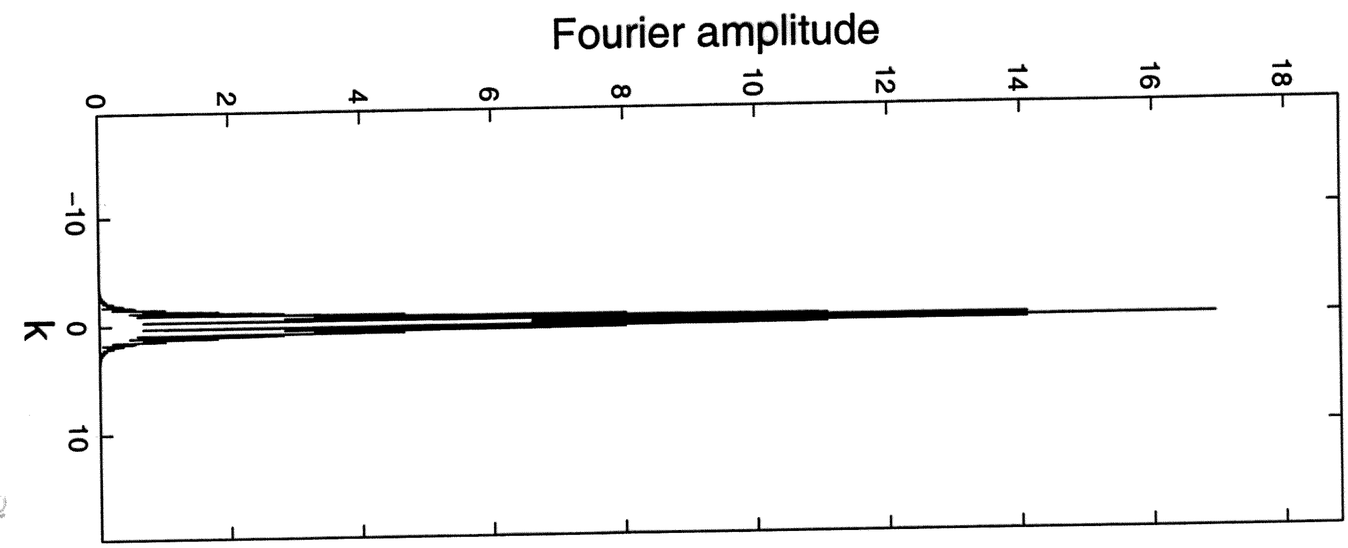

Fourier amplitude
0
1
2
3
4
5
6
7
8
9
10
-10
0
10
k

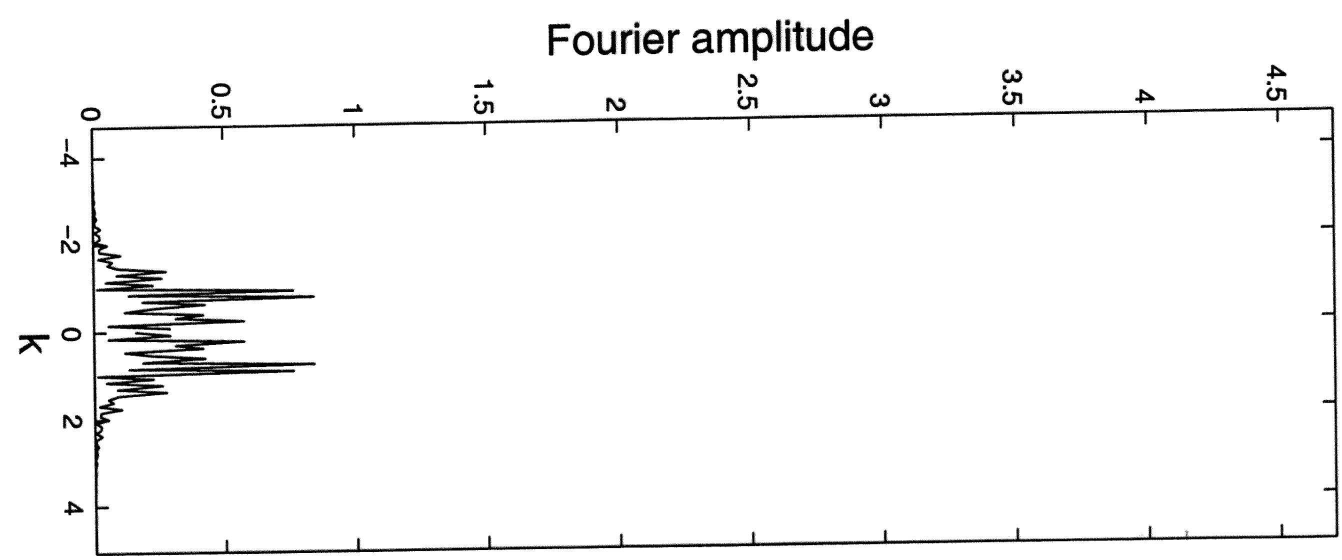

Fourier amplitude
0
0.2
0.4
0.6
0.8
1
1.2
-2
-1
0
1
2
k

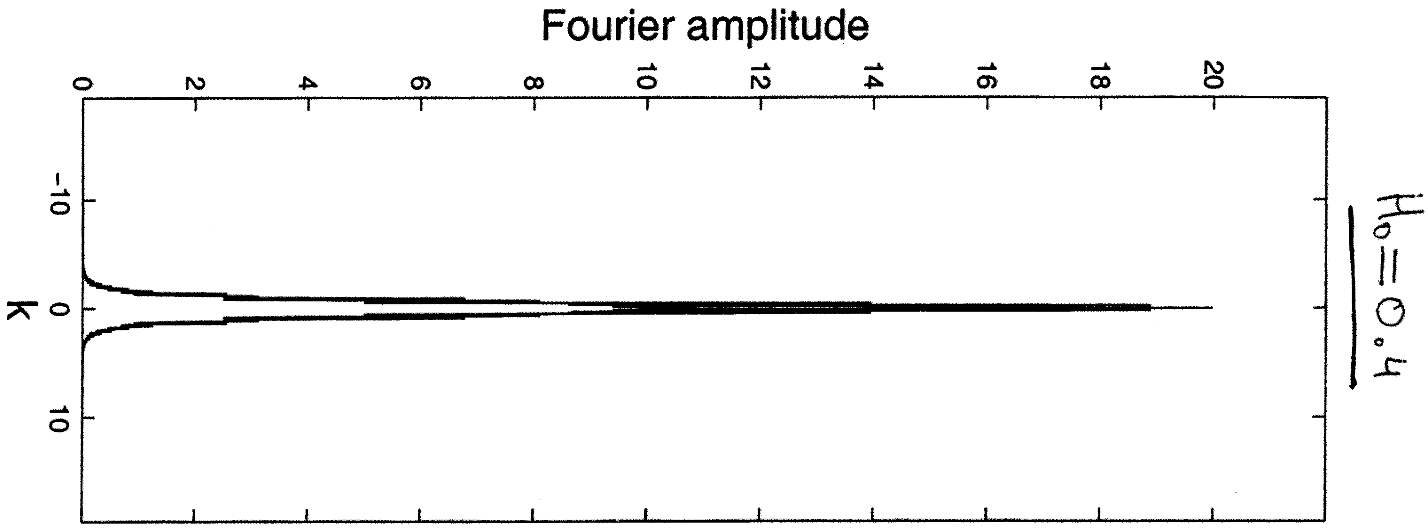

Fourier amplitude
0
5
10
15
-10
0
10
k
$H_0 = 0.2$
Fourier amplitude
0
5
10
15
-10
0
10
k
Fourier amplitude
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
-2
-1
0
1
2
k
Ph. Guyenne (2004)

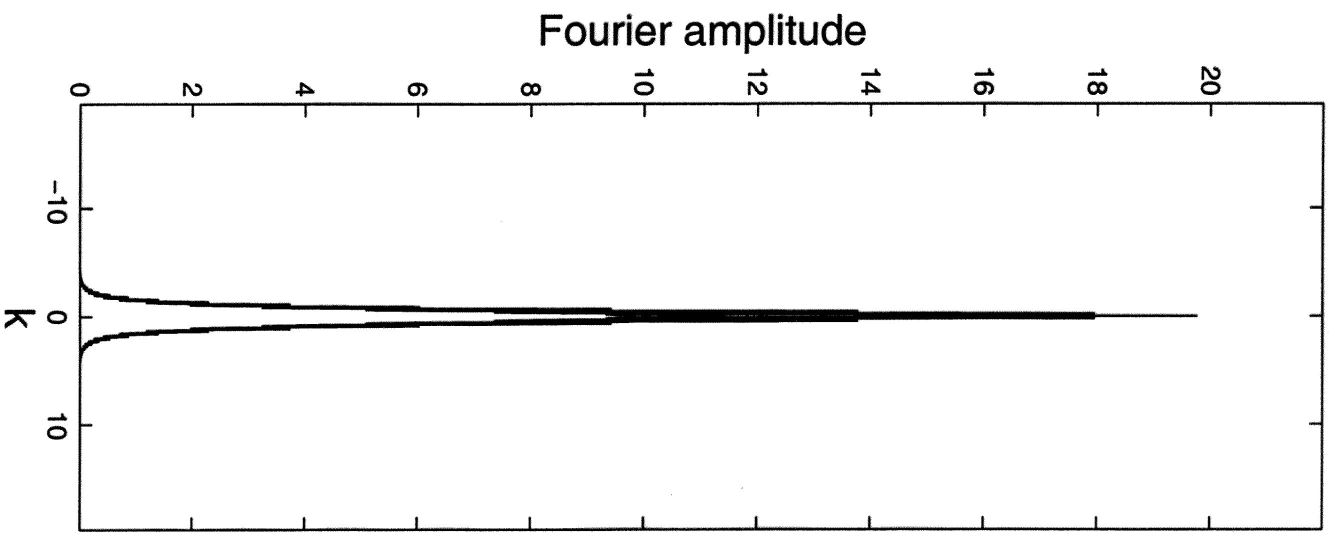
$H_0 = 0.3$

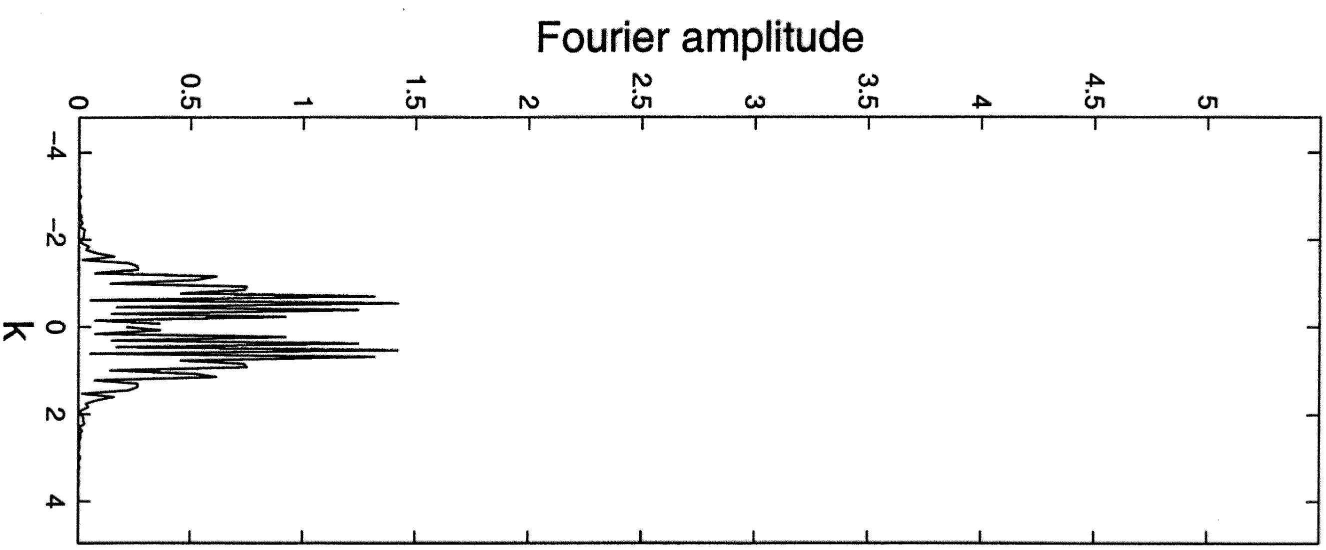

Fourier amplitude
0 2 4 6 8 10 12 14 16 18
k
-10 0 10


Fourier amplitude
0 2 4 6 8 10 12 14 16 18
k
-10 0 10


Fourier amplitude
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5
k
-4 -2 0 2 4


Fourier amplitude
k
H0 = 0.4


Fourier amplitude
k

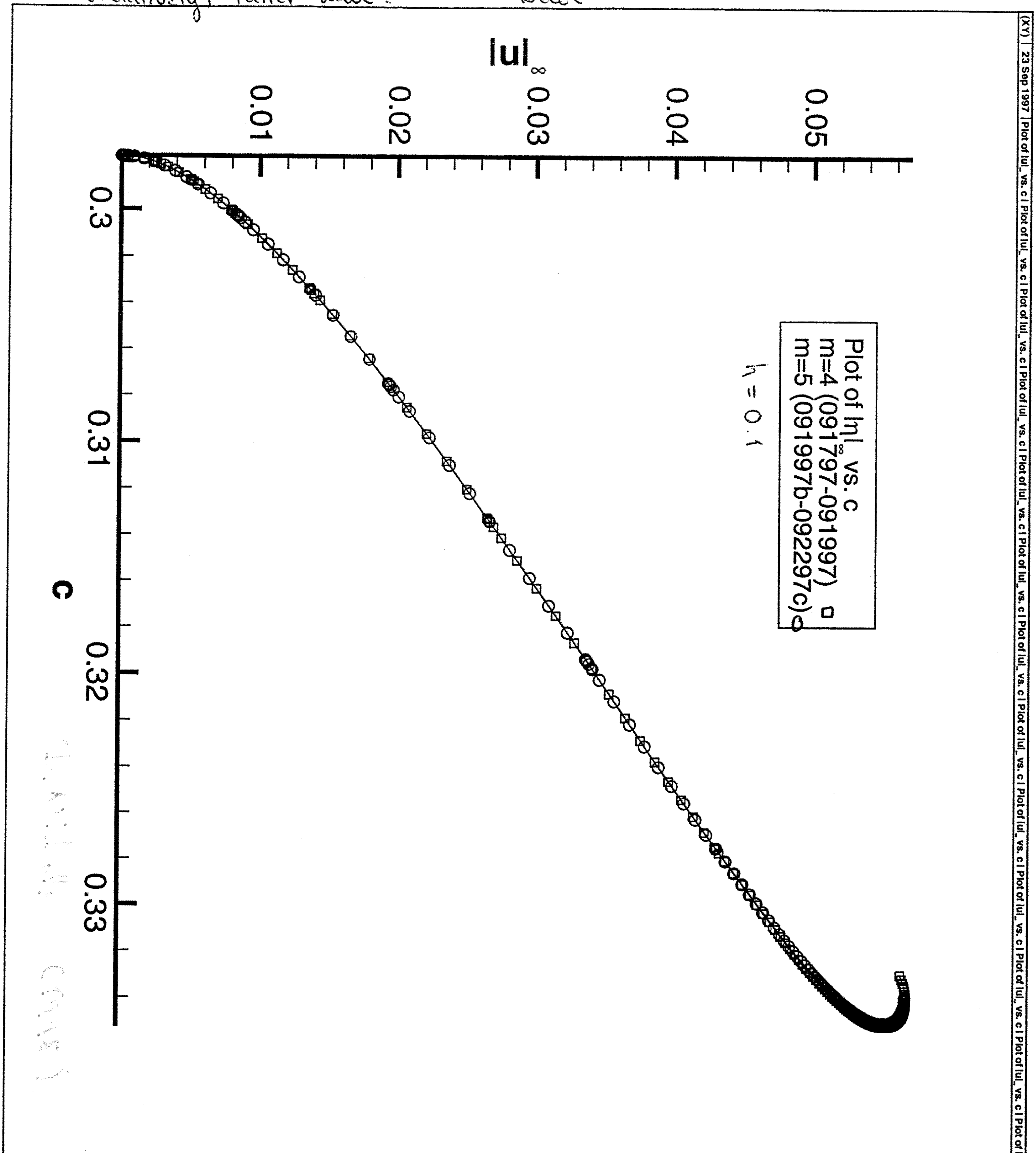

Fourier amplitude
k

9/23/97

Walter,

Here are my results for the m=5 runs with h = 1/10. Again N=512. Now I'm trying h = 1/100 to see if I can get a (relatively) taller wave.

Dave



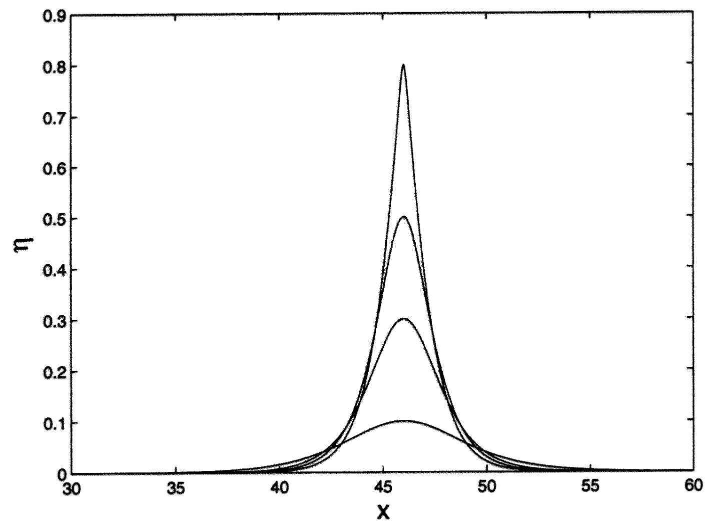
Solitary waves of Euler equations

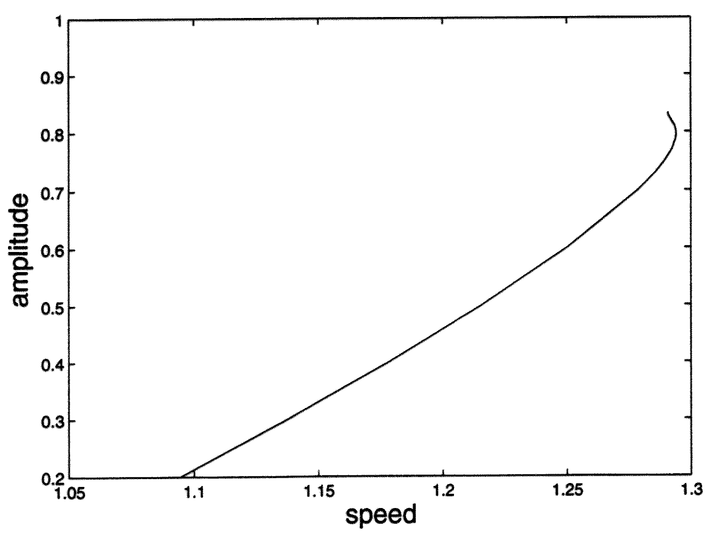
- Tanaka's method (1986):

- Boundary integral formulation, Cauchy's theorem

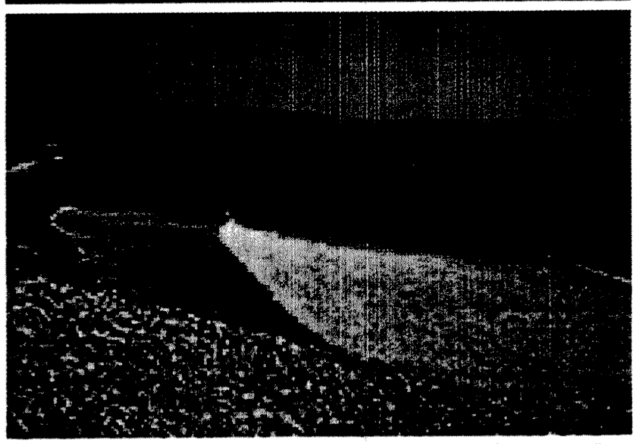
- Finite difference schemes, non-uniform grid

- Solution of nonlinear equations by fixed pt. iteration





Solitary waves in nature



Heriot-Watt 1995