A BEPU ANALYSIS SEPARATING EPISTEMIC AND ALEATORY ERRORS TO COMPUTE ACCURATE DRYOUT POWER UNCERTAINTIES

REACTOR SAFETY

KEYWORDS: *best estimate plus uncertainty analysis, epistemic error and aleatory variation, confidence and tolerance intervals*

D. PUN-QUACH,[a] P. SERMER,[a] F. M. HOPPE,[b]* O. NAINER,[c] and B. PHAN[d]

[a]*AMEC NSS Limited, Toronto, Ontario, Canada*
[b]*McMaster University, Hamilton, Ontario, Canada*
[c]*Bruce Power LP, Toronto, Ontario, Canada*
[d]*Ontario Power Generation, Inc., Toronto, Ontario, Canada*

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This paper presents a best estimate plus uncertainty (BEPU) methodology applied to dryout, or critical channel power (CCP), modeling based on a Monte Carlo approach. This method involves the identification of the sources of uncertainty and the development of error models for the characterization and separation of epistemic and aleatory uncertainties associated with the CCP parameter. Furthermore, the proposed method facilitates the use of actual operational data leading to improvements over traditional methods, such as sensitivity analysis, which assume parametric models that may not accurately capture the possible complex statistical structures in the system input and responses.

I. INTRODUCTION

Best estimate (BE) plus uncertainty (BEPU) methods are widely used for nuclear safety analyses.[1–6] These methods use realistic codes to represent the physical phenomena that underlie the safety analyses. The use of BE codes within the reactor technology, either for design or safety analysis, requires an understanding of the limitations and deficiencies associated with these codes.

It has been shown in Refs. 7 through 10 that a methodology utilizing properties of extreme value statistics (EVS) (the so-called EVS methodology) can lead to significant improvements in probabilistic safety analyses (PSAs). A critical component of the EVS methodology utilizes the distinction between aleatory and epistemic uncertainties in the evaluation of the safety margins. In the nuclear risk analysis, aleatory uncertainty reflects the many hypothetical accident scenarios that are considered in the respective risk analysis. "Aleatory" is often used synonymously with "random" and in the EVS methodology refers to variations in underlying conditions whose effects cannot be predicted in advance and that result from a variety of operating conditions or states. These arise because the reactor core is subject to continuous change, such as refueling, operation of the reactor regulating system, changes in thermal-hydraulic conditions, inter alia. The timing and interaction of these changes cannot be predicted in advance, and this results in the state of the core being subject to random variations. The term "epistemic" refers to the state of, or lack of, knowledge of the underlying physical phenomena, and an epistemic error refers to the difference between what a BE code or measurement is attempting to measure and the value actually obtained. Epistemic uncertainties arise because of the many uncertain parameters that underlie the estimation of the probabilities and consequences of the respective hypothetical accident scenarios, including, for example, the inaccuracy of computer codes as well as the inaccuracy of both the variables that are input into these codes and the inaccuracy of measurements that characterize different aspects of station operation. Epistemic error results in a perception of the reactor state differing from the true reactor state.

It is well recognized that a clear discrimination between these uncertainties is a requisite for decision making in the environmental risk and safety assessment

*E-mail: hoppe@mcmaster.ca

industry.[11,12] For example, Apostolakis[12] shows how separating aleatory and epistemic uncertainties can make a difference in PSA using an example involving the modeling of pipe failures under plant aging effects. In a similar fashion, separation of epistemic and aleatory errors arises naturally in the EVS methodology. Aleatory error determines a (lower) percentile of an unknown distribution of trip setpoints. The estimate of this percentile must be based on variables that are observable/computable, and these are based on variables that are subject either to have measurement error or to be BEs obtained using imperfect computer codes, hence subject, in each case, to epistemic error. In the simplest context, we would have $Y_i = X_i + E_i$, $1 \leq i \leq n$, where X_i are aleatory random variables each representing a realization of a quantity of interest, a percentile of whose distribution is to be estimated; Y_i are the corresponding observed/computed values (either measured or code computed), and E_i are the epistemic errors. The Y_i are then used to draw an inference on the specified percentile of the X distribution. Thus, it is natural to separate out the two types of uncertainty, and despite different origins, both epistemic and aleatory variations can be analyzed within the same probabilistic framework to derive a 95/95 tolerance interval.

These results presented have led to significant improvements in the evaluation of the safety margins when applied to the neutron overpower protection (NOP) trip setpoint analysis[7,8,10] for demonstrating shutdown system effectiveness. An important input in the NOP analysis is the calculation of the critical channel powers (CCPs) and uncertainties associated with the CCP parameter. The calculations of the CCPs rely on BE thermal-hydraulic codes to model the response of the heat transport (HT) system (HTS) under a postulated slow loss of regulation (LOR) event considered in the NOP analysis.

The objective of this paper is to describe a BEPU analysis using a Monte Carlo approach. The paper describes the methodology for the mathematical modeling of the input and response variables of CCPs that reflect the different sources of code-related uncertainties and distinguishes between epistemic and aleatory errors. A statistical framework that provides the integration of the mathematical error models for the evaluation of the 95/95 tolerance interval based on the EVS methodology is also given in this paper. The statistical framework is utilized in demonstrating the required nuclear safety margins associated with the accident analysis.[7]

II. BACKGROUND

II.A. NOP Trip Coverage Analysis and the Underlying Statistical Framework

During a postulated power excursion, such as a LOR event, the reactor power may increase sufficiently to induce an unstable dry patch on the fuel sheath in a high-power channel. This condition is commonly known as dryout.[a] Although the onset of intermittent dryout of the fuel sheath does not necessarily lead to fuel or fuel channel failures, elevated fuel temperatures can result in fuel element deformations and, possibly, fuel centerline melting and eventually pressure tube (PT) failure. Thus, the prevention of the onset of intermittent dryout has been used in the Canadian safety analysis industry as a conservative criterion for preventing fuel failures leading to radiological releases.

The NOP system is based on a margin-to-trip approach that is a function of the NOP trip setpoint and calibration-related factor. A challenging aspect associated with the determination of the NOP safety margins is that the input variables required in the NOP analysis may not be directly measurable quantities but are computed using complex BE codes, including determination of the dryout power or CCP that is used in assessing the margin to dryout. The CCP calculations rely on a thermal-hydraulic code that is a function of input variables that define the initial boundary conditions (e.g., reactor header conditions, etc.) and those that define the properties and phenomena of the system and code (e.g., flow area of the channel, etc.).

A statistical framework to evaluate the operational margins based on a 95/95 tolerance interval has been proposed in Ref. 7 and utilizes the EVS methodology. The results of the EVS methodology have demonstrated improvements in the operational safety margins associated with the NOP trip setpoint analysis. As discussed in Ref. 7, the statistical framework for computing an NOP trip setpoint is based on a mathematical model that leads to Eqs. (1) and (2):

$$t = t^0 + \Theta \tag{1}$$

and

$$T = t^0 + \tau \ , \tag{2}$$

where t describes a "true" trip setpoint, which in the NOP analysis refers to the trip setpoint at the precise moment that the NOP system initiates a reactor shutdown to prevent (intermittent) dryout in any one of the reactor fuel channels. The reactor state at such future time will not be known and is modeled as an aleatory variable. The variable t^0 in Eq. (1) corresponds to a reference trip setpoint at a specified set of thermal-hydraulic conditions of interest in the NOP analysis. However, t^0 can still depend on non-thermal-hydraulic variables. Different reference conditions are considered in the evaluation of the NOP trip margins to reflect the changes in the thermal-hydraulic characteristics of the HTS as the reactor core ages. These aging effects include steam generator tube fouling, changes in feeder

[a] The channel power at which dryout occurs in a channel is known as the CCP.

roughness due to flow-accelerated corrosion, and nonuniform physical changes in the fuel channels due to irradiation-enhanced creep and hoop stress. These aging effects lead to changes in the reactor header conditions, core flow, thermal-hydraulic properties of the fluid, and HT capabilities leading to potential degradation in the available margins. The variations in these thermal-hydraulic conditions are reflected in the quantity Θ. At the given reference conditions, the BE codes are used to compute T, an estimate of t^0 with an error denoted by τ in Eq. (2).

The quantity τ is an epistemic error based on our earlier definition. As a result, the BE value T is considered a random variable. This may require clarification since the BE codes used are deterministic (i.e., running a code produces a single output and will always produce the same value under the same conditions). We are assuming that such a single output is only a random realization of a multitude of values that could be obtained under different realizations of the errors that are inherent in the code or the input variables that are themselves measurements or computed by some other codes.

The variable Θ in Eq. (2) is also considered a random variable and is considered aleatory insofar as it reflects random variations in true variables. The concept "true" may sometimes represent only our mathematical perception of truth to the extent that we are able to model the true nature of the underlying physical process. This "generalization" of "truth" allows us to characterize the errors rigorously (we will see an example in this paper). Random variations in t in Eq. (1) are reflected in Θ. However, these are not the only possible aleatory variations in t. As pointed out above, the reference variable t^0 may also depend on other variables that may be considered random insofar as they happen in the future under postulated LOR events at initial conditions that cannot be known a priori. An example would be the channel powers under normal operating conditions that vary due to on-line fueling and the reactor regulating system that ensures that the operations are within the safe operating envelope.

We note that Eqs. (1) and (2) describe a rather general mathematical model that can be applied to other safety or compliance problems. Equations (1) and (2) represent a model that explicitly differentiates between epistemic and aleatory variations. This distinction leads to a statistical decision problem in BEPU-type analyses, which allows us to generalize a notion of a tolerance limit to epistemic uncertainty. For instance, we may define such a lower tolerance limit W by

$$P[W \leq t_{1-\gamma}] = \beta \ , \tag{3}$$

where P is the probability defined over the space of data that define the problem given by Eqs. (1) and (2). The novelty is that $t_{1-\gamma}$ is a parameter determined by aleatory conditions, while P is evaluated with respect to the epistemic uncertainty since the random W arises from a code and measurements. Note that γ and β define the level of safety for the decision-making problem where $t_{1-\gamma}$ is the lower $100(1-\gamma)$ percentage point of the probability distribution for t and $1-\gamma$ typically represents the risk that the regulator and utilities agree is tolerable.[13] Furthermore, β is the confidence level (typically taken to be 0.95), and W results from the presence of epistemic error in Eq. (2) together with the aleatory variation in Eq. (1) resulting from a finite data set. Without the epistemic component, W would reduce to the usual tolerance interval as in Ref. 14. The solution to Eq. (3) requires the construction of a random variable W that is the result of code and measurement and whose probability distribution provides a one-sided lower confidence interval for the aleatory parameter $t_{1-\gamma}$. To our knowledge, this is the first instance of a frequentist tolerance interval involving epistemic error.

The solution of the problem defined by Eqs. (1), (2), and (3), the computed tolerance limit W, will serve as the required trip setpoint. The reason for taking the lower tolerance limit is the result of the NOP system requirement of tripping the reactor before the onset of dryout with a high level of assurance.

On the other hand, for a different problem, such as loss-of-coolant accident, where the decision-making problem is dependent on fuel temperature, the appropriate limit to compute is the upper tolerance limit given by $P[t_{\gamma} \leq W] = \beta$. Construction of the random variable W is beyond the scope of the present paper and can be found in Ref. 15. What is important, for the purpose of this paper, is that the error distributions that are needed for W are obtained by the methodology described here. However, for the interested reader, the ideas behind the construction of W are as follows. For a lower limit, we apply the Central Limit Theorem to the multiple observations $\{T_1, \ldots, T_n\}$ described by Eq. (2) to obtain a random variable $\overline{V} - \lambda S_V$ where $\overline{V}$ is a centered average of the T_i, S_V is the sample standard deviation of the V_i, and the constant λ is to be determined in order to achieve the specified confidence β. For large sample size, we can obtain a relationship between percentiles of the two random variables on the left sides of Eqs. (1) and (2) and then estimate λ. The random variable W is then given as $W = \overline{V} - \lambda S_V$. In these forms, W resembles the form of the usual tolerance interval but now in a context involving epistemic error.

II.B. CANDU Reactor Design

The error analysis methodology discussed in this paper was implemented for an actual nuclear power plant, briefly described here because it differs from the more commonly used pressurized water reactor. The plant of interest, Bruce Power's nuclear generation station (NGS), is based on a CANada Deuterium Uranium (CANDU) reactor design, which is a pressurized heavy water reactor. Heat removal from the fission process is accomplished in

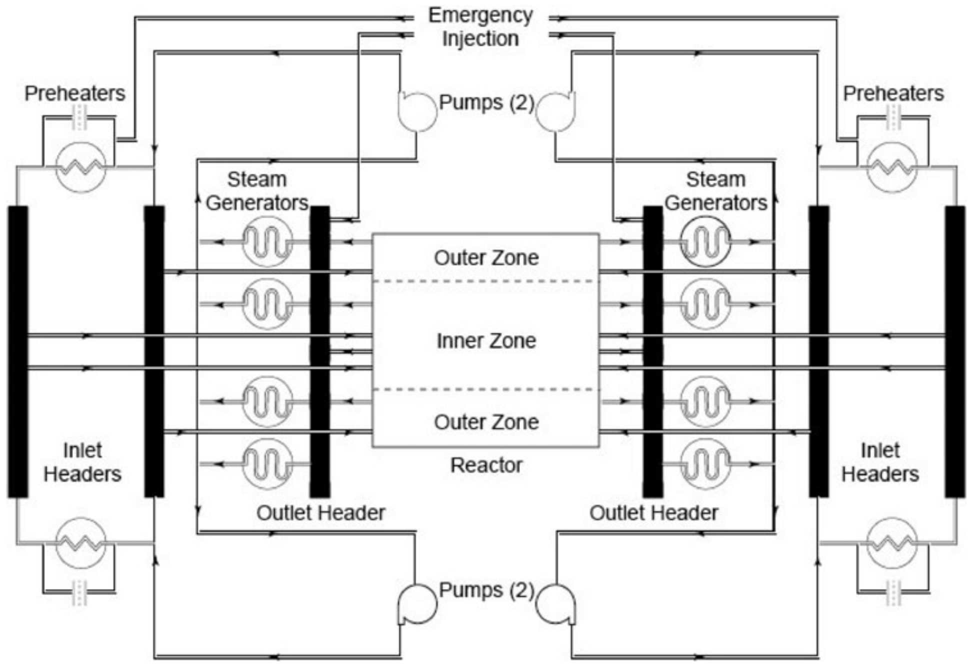

Fig. 1. Bruce NGS CANDU reactor with inner and outer thermal-hydraulic flow zones (images taken from Ref. 16).

a CANDU reactor through a HTS as given in Fig. 1. The HTS accomplishes the safety-related goal of cooling the fuel. The complete flow pattern of the HTS resembles that of a figure eight.

A particularly unique HTS is that of the Bruce NGS's design where the HTS is a single closed loop but the core is physically divided into two separate hydraulic flow zones, referred to as the outer zone (OZ) and the inner zone (IZ) (see Fig. 1). Hot coolant flow in the loop passes through the boilers, removing heat and reducing the coolant temperature, and then passes through the HT pumps, adding pump head to the pressure. After the HT pumps, the flow splits into the two flow zones. The fuel channels in the OZ are connected to a single reactor inlet header (RIH) on each side of the loop (one east and one west) only. The portion of the coolant flow that goes to the OZ goes directly to the OZ RIH at the boiler outlet temperature and pressure after the HT pumps and is completely separate from the IZ. There is a total of 480 fuel channels at Bruce NGS.

The fuel channels in the IZ are connected to a single RIH on each side of the loop (one east and one west) only, separate from the OZ RIH. The portion of the coolant flow that goes to the IZ does not go directly to the IZ RIH but first flows through a preheater heat exchanger that removes more heat from this portion of the coolant, further reducing the IZ coolant temperature. The flow goes from the preheater to the IZ RIH and is completely separate from the OZ. The fuel channels in the IZ therefore experience flow conditions coming into the channels that are lower in temperature due to heat removal by the preheater and also at a lower pressure as some pressure is lost as coolant passes through the preheater, relative to the fuel channels in the OZ. The two zones join together downstream of the fuel channels and before the boilers, via a reactor outlet header on each side of the loop.

III. THE BEPU CCP UNCERTAINTY ANALYSIS

III.A. Overall Approach

Estimates of dryout power for each fuel channel in a reactor core are computed using the TUF thermal-hydraulic code,[17] through a series of iterative steady-state thermal-hydraulic calculations. The initial boundary conditions and bundle power distributions corresponding to the LOR event are used to calculate the channel flow and thermal-hydraulic conditions along the channel. Based on the local thermal-hydraulic conditions, the critical heat flux (CHF) at each axial node is determined and compared against the axial heat flux. The computed channel power is increased until the CHF profile becomes tangential to the axial heat flux profile, which occurs at the CCP (i.e., the channel power required to induce intermittent dryout).

A significant component of the application of a BEPU method for the purpose of nuclear safety analysis is the identification and characterization of uncertainties related to the BE codes. Several international activities exist that summarize the current state-of-the-art methods for evaluating the uncertainties associated with the prediction of computer codes in nuclear safety technology.[2,18–20] Efforts to characterize the different "sources of uncertainty" that affect the predictions of the BE codes have been completed in Ref. 20. In a broad sense, these sources of uncertainty are categorized as follows:

1. code or model related uncertainty
2. representation uncertainties
3. scaling related issues
4. uncertainties in the measurements
5. user effects.

The CCP BEPU analysis discussed in this paper explicitly considers these different sources of uncertainty and is implemented using a Monte Carlo method for the purpose of error propagation of the input variables. In addition, the methods based on data assimilation consistent with that in Refs. 21, 22, and 23 are utilized to reflect code-related uncertainties in items 1 and 2 above. This method utilizes results and data from experimental, commissioning, and actual operating station data. The results of the data assimilation methods and statistics used in this BEPU analysis ensure that the parameters and the predicted responses minimize the representation uncertainties of the system under study.

The uncertainty in the code or model exists since the system thermal-hydraulic code is a computational tool that typically includes three different sets of balance equations (i.e., energy, mass, and momentum), closure or constitutive equations, material and state properties, and a numerical solution method. In addition, parameters that define the boundary initial conditions may rely on measurements from plant instrument data or other BE codes. Empirical models are widely used to "substitute" the balance or governing equations to describe these physical phenomena implemented within the computational code. The empirical models being based on experimental or measured data are subject to error. The scope of this paper focuses on presenting a methodology for mathematically modeling the epistemic and aleatory errors in both the inputs and response variable of a BEPU Monte Carlo analysis. The BEPU Monte Carlo approach used in evaluating and modeling the CCP error is discussed in Secs. III.B and III.D, respectively. The error modeling method for the input variables of the CCP BEPU analysis is described in Sec. III.C. The data assimilation methodology implemented as part of the BEPU analysis is not discussed in detail in this paper, but it will be examined in future papers.

III.B. A Monte Carlo Method for the CCP Error Models

The Monte Carlo methodology for evaluating the uncertainties in CCPs can be described as follows. Let

$$\boldsymbol{x} = (x_1, x_2, \ldots, x_p)^T \tag{4}$$

be a vector representing the important random process and modeling variables that determine the CCPs, $\boldsymbol{ccp} = (ccp_1, ccp_2, \ldots, ccp_N)$, where N is the total number of fuel channels (such as 480 in Bruce Power NGS reactors). The superscript T means transpose and anticipates the algebra used in the Appendix, where $\boldsymbol{x}$ will be represented as a column vector. Both $\boldsymbol{x}$ and $\boldsymbol{ccp}$ are understood to be the actual, or true, variables related by

$$\boldsymbol{ccp} = \boldsymbol{g}(\boldsymbol{x}) \ . \tag{5}$$

The function $\boldsymbol{g}$ represents the perfect understanding of the system (i.e., prediction of power required to induce intermittent dryout) and is merely introduced as a convenient way to show the variables that $\boldsymbol{ccp}$ depends on. The input (vector-valued) variable $\boldsymbol{x}$ in Eq. (4) is assumed to be a random variable that is centered about some reference value $\boldsymbol{x}^0$. Thus, we may write

$$\boldsymbol{x} = \boldsymbol{x}^0 + \boldsymbol{\vartheta} \ , \tag{6}$$

where $\boldsymbol{\vartheta}$ is an aleatory variable (sometimes referred to as "error") that reflects the variations in $\boldsymbol{x}$; $\boldsymbol{x}^0$ may depend on other variables that also may be random providing additional aleatory variation from a different source.

Components of $\boldsymbol{x}$ represent true input values and are therefore unknown. These variables are estimated either by measurements or by available physics or thermal-hydraulic codes. These estimates will be denoted by $\boldsymbol{X} = (\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_p)^T$ and are assumed to approximate $\boldsymbol{x}^0$ with an error denoted by $\boldsymbol{\varepsilon}$ as shown by

$$\boldsymbol{X} = \boldsymbol{x}^0 + \boldsymbol{\varepsilon} \ . \tag{7}$$

Recall that $\boldsymbol{x}^0$ also contains modeling parameters and the corresponding components in $\boldsymbol{\varepsilon}$ reflect code errors. Methods based on data assimilation[21–23] are utilized to capture the station-specific phenomena and properties such as those described in Fig. 1. Since the code errors are presumed to be part of the input errors (the so-called backward error analysis[24]), we will use $\boldsymbol{g}$ in Eq. (5) to denote the thermal-hydraulic code (described in Sec. III.A), and the computation of $\boldsymbol{CCP}$ can be described as

$$\boldsymbol{CCP} = \boldsymbol{g}(\boldsymbol{X}) \ , \tag{8}$$

which are referred to as the BE values of $\boldsymbol{ccp}$ at the reference conditions. We note that the input error model Eqs. (6) and (7) follows the same structure as the one provided by Eqs. (1) and (2). The equation $\boldsymbol{CCP} = \boldsymbol{g}(\boldsymbol{X})$ should, more strictly, be written as $\boldsymbol{CCP} = \boldsymbol{g}^*(\boldsymbol{X})$; however, any differences between the hypothetical function $\boldsymbol{g}$ and the actual $\boldsymbol{g}^*$ that utilizes code and measurements are absorbed in the epistemic error term $\boldsymbol{\varepsilon}_{\boldsymbol{ccp}}$ in Eq. (10). As pointed out earlier, in this paragraph this substitution is part of the backward error analysis where differences between $\boldsymbol{g}$ and $\boldsymbol{g}^*$ are absorbed in $\boldsymbol{\varepsilon}_{\boldsymbol{ccp}}$.

As described earlier, the errors $\boldsymbol{\varepsilon}$ in Eq. (7) are epistemic in nature. We assume that the distribution of $\boldsymbol{\varepsilon}$ is known or could be evaluated based on the available validation data. Similarly, the distribution of the error ϑ in Eq. (6) is assumed known or estimable based on reactor operation data.

With the above definitions, we are now in a position to derive an error model for the CCP variable. Naturally, we will seek such a model in the form as given by Eqs. (1) and (2) or Eqs. (6) and (7). That is,

$$\boldsymbol{ccp} = \boldsymbol{ccp}^0 + \boldsymbol{\vartheta}_{ccp} \tag{9}$$

and

$$\boldsymbol{CCP} = \boldsymbol{ccp}^0 + \boldsymbol{\varepsilon}_{ccp} \ , \tag{10}$$

where

$$\boldsymbol{ccp}^0 = \boldsymbol{g}(\boldsymbol{x}^0) \ .$$

The errors $\boldsymbol{\vartheta}_{ccp}$ and $\boldsymbol{\varepsilon}_{ccp}$ can be obtained from

$$\begin{aligned}\boldsymbol{\vartheta}_{ccp} &= \boldsymbol{ccp} - \boldsymbol{ccp}^0 = \boldsymbol{g}(\boldsymbol{x}) - \boldsymbol{g}(\boldsymbol{x}^0)\\ &= \boldsymbol{g}(\boldsymbol{x}^0 + \boldsymbol{\vartheta}) - \boldsymbol{g}(\boldsymbol{x}^0)\end{aligned} \tag{11}$$

and

$$\begin{aligned}\boldsymbol{\varepsilon}_{ccp} &= \boldsymbol{CCP} - \boldsymbol{ccp}^0 = \boldsymbol{g}(\boldsymbol{X}) - \boldsymbol{g}(\boldsymbol{x}^0)\\ &= \boldsymbol{g}(\boldsymbol{x}^0 + \boldsymbol{\varepsilon}) - \boldsymbol{g}(\boldsymbol{x}^0) \ .\end{aligned} \tag{12}$$

Equations (11) and (12) show how Eqs. (1) and (2) arise.

Estimates of $\boldsymbol{\varepsilon}_{ccp}$ and $\boldsymbol{\vartheta}_{ccp}$ are required inputs into the NOP trip setpoint calculations. Equations (11) and (12) lend themselves to computing the probability distributions for $\boldsymbol{\varepsilon}_{ccp}$ and $\boldsymbol{\vartheta}_{ccp}$ using a Monte Carlo numerical simulation by sampling from the known distributions for $\boldsymbol{\varepsilon}$ and $\boldsymbol{\vartheta}$. The difficulty is that $\boldsymbol{x}^0$, being a true quantity, is unavailable. Following the prescription in Ref. 25, we apply a surrogate approach in which we substitute the unknown $\boldsymbol{x}^0$ with the BE value $\boldsymbol{X}$. This is a reasonable approach provided that $\boldsymbol{X}$ is a sufficiently close estimate of $\boldsymbol{x}^0$. In other words,[13] it is required that the code being used in the Monte Carlo simulation needs to be phenomenologically correct. Thus, we obtain

$$\boldsymbol{\vartheta}^s_{ccp} = \boldsymbol{g}(\boldsymbol{X} + \boldsymbol{\vartheta}) - \boldsymbol{g}(\boldsymbol{X}) \tag{13}$$

and

$$\boldsymbol{\varepsilon}^s_{ccp} = \boldsymbol{g}(\boldsymbol{X} + \boldsymbol{\varepsilon}) - \boldsymbol{g}(\boldsymbol{X}) \tag{14}$$

as approximations to $\boldsymbol{\vartheta}_{ccp}$ and $\boldsymbol{\varepsilon}_{ccp}$, respectively; that is, the estimated distribution for $\boldsymbol{\vartheta}^s_{ccp}$ is used in Eq. (1) to estimate the distribution for Θ, and the estimated distribution for $\boldsymbol{\varepsilon}^s_{ccp}$ is used in Eq. (2) to compute the distribution for τ. These errors $\boldsymbol{\vartheta}^s_{ccp}$ and $\boldsymbol{\varepsilon}^s_{ccp}$ are used in a Monte Carlo simulation to obtain the tolerance limit W required in Eq. (3). Therefore, any dependencies among the errors need to be known to enable proper sampling in the simulation, yet Eqs. (13) and (14) do not explicitly reveal such dependencies. In Sec. III.D we show how to derive the error structure for $\boldsymbol{\vartheta}^s_{ccp}$ and $\boldsymbol{\varepsilon}^s_{ccp}$ with the explicit error distributions.

III.C. Mathematical Modeling of the Epistemic and Aleatory Errors in the CCP Input Variables

As discussed in Sec. III.A, the CCPs are calculated using a BE thermal-hydraulic code. The reactor header condition and bundle power distribution corresponding to the LOR event are used to calculate the channel flow and thermal-hydraulic conditions along each channel. Based on these thermal-hydraulic conditions, the CHF at each axial node is determined and compared against the axial heat flux corresponding to the accident conditions. The computed channel power is increased until the CHF profile becomes tangential to the axial heat flux profile. Empirical models are required to provide the necessary inputs in calculating the CCPs. The empirical models include those that are used to compute the CHF, fuel channel geometry, and the calculation of the pressure losses for each component of the flow path from header-to-header along a fuel channel. Using a regression analysis, these empirical models are developed and utilize full-scale experimental data and/or operational station measurements. It is understood that there exist a number of variables that must be considered as inputs into the CCP BEPU analysis with varying levels of complexity. However, this section provides an illustrative example that provides a means to demonstrate how one distinguishes and characterizes the epistemic and aleatory errors associated with the input variable of interest. The results are expressed in the required form as described by Eqs. (6) and (7). The methodology can easily be extended to other input variables.

The methodology presented here is for the development of the BE prediction and error modeling of PT diameters for each fuel channel. The nonuniform change in PT dimensions is a principal aging mechanism governing the HT and hydraulic degradation of the HTS. For the purpose of demonstration, the prediction of PT diameters for bundle i, channel j can be developed using measured PT diameters (or measured strain[b]) and a linear functional model that expresses its dependency to fluence ψ (and integrated fuel irradiation over time) as well as the (lifetime-averaged) coolant temperature ω.

The linear functional form of PT strain is given by

$$s_{ij} = a_j + b_i \psi_{ij} + c_i \omega_{ij} + \delta_j \ , \tag{15}$$

where

$$s^o_{ij} = a_j + b_i \psi_{ij} + c_i \omega_{ij} \ , \tag{16}$$

and where

s^o_{ij} = functional form of the strain

δ_j = aleatory error.

[b] Note that measured strain S_{ij} is defined as $(D_{ij} - D_o)/D_o$, where D_{ij} and D_o are the measured and reference PT diameters at bundle i, channel j, respectively.

It is assumed that Eq. (15) represents a true value of the PT strain. This, of course, cannot be literally true. However, it is a reasonable model, and we assume that the amount by which it is simplified is negligible with respect to all other variables that may be present (this assumption is substantiated using existing measurement). In the absence of a physical model, we determine the coefficients a_j, b_i, and c_i, using regression, and the existing measurements S_{ij}. The difficulty here is that the measurements are available only on a sample of all PTs, and therefore, a_j cannot be determined for all the PTs. Assuming that a_j reflects differences among the PTs due to manufacturing uncertainties or material properties, we will model the strain as a random variable, and the channels that have measurements available will be considered a random sample from the population of all PTs (note that in the statistical jargon, such models are described as "random effects models"). Equation (16) can be written compactly in matrix notation as

$$\boldsymbol{s}^0 = \mathbf{A}\boldsymbol{\beta} \ , \tag{17}$$

where

$\boldsymbol{\beta}$ = vector of the coefficients a, b_i, and c_i

$\mathbf{A}$ = data matrix consisting of $\psi_{i,j}$ and $\omega_{i,j}$.

Likewise, in matrix notation, Eq. (15) can be expressed as

$$\boldsymbol{s} = \boldsymbol{s}^0 + \mathbf{B}\delta \ . \tag{18}$$

Note that entries in matrix **B** are only zeros and ones to preserve a proper ordering reflecting the arrangement in Eqs. (16) and (17). Observe that Eq. (18) is exactly of the same form as Eqs. (1), (6), or (9) with the error structure made explicit.

The epistemic part of the PT diameter model is derived by fitting Eq. (18) to the available measurements. Let those measurements be denoted by $S_{i,j}$, and correspondingly, let the vector of all measurements [consistent with the index arrangement in Eq. (17)] be denoted by $\boldsymbol{S}$. If the measurement error (epistemic by definition) is denoted by ξ_{ij}, or, $\boldsymbol{\xi}$ in the vector notation, then

$$\boldsymbol{S} = \mathbf{A}\boldsymbol{\beta} + \mathbf{B}\boldsymbol{\delta} + \boldsymbol{\xi} \tag{19}$$

is the random effects regression model. We will assume that the error components of $\boldsymbol{\delta}$ and $\boldsymbol{\xi}$ are normally and independently distributed, such that

$$\delta_j \sim \mathcal{N}(0, \sigma_\delta^2)$$

and

$$\xi_{ij} \sim \mathcal{N}(0, \sigma_\xi^2) \ .$$

We will show how to solve the above model in the Appendix [using maximum likelihood estimation (MLE)]. If the term $\mathbf{B}\boldsymbol{\delta}$ in Eq. (19) were missing, then the resulting regression model would be a standard regression that is solvable by the usual techniques of least squares. However, the presence of $\mathbf{B}\boldsymbol{\delta}$ makes Eq. (19) a problem with a rather complex error structure, and special care is needed to solve it. Since the variances σ_δ^2 and σ_ξ^2 are unknown parameters, the problem becomes nonlinear.

Let the MLE solution of the coefficients $\boldsymbol{\beta}$ in Eq. (19) be denoted by $\hat{\boldsymbol{\beta}}$. Thus, the BE strain (or the "code" prediction) is denoted by $\hat{S}$ and is given by

$$\hat{S} = \mathbf{A}\hat{\boldsymbol{\beta}} \ , \tag{20}$$

where the matrix **A** is evaluated at such data for which the prediction of $\boldsymbol{s}^0$ is required. The error $\boldsymbol{\eta}$, between the BE $\hat{S}$ and the true value $\boldsymbol{s}^0$, is given by

$$\boldsymbol{\eta} = \hat{S} - \boldsymbol{s}^0 = \mathbf{A}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) \ .$$

Thus, the epistemic model for the BE value is

$$\hat{S} = \boldsymbol{s}^0 + \boldsymbol{\eta} \ . \tag{21}$$

The properties of the error $\boldsymbol{\eta}$ are derived in the Appendix. In particular, it is shown that $\boldsymbol{\eta}$ is unbiased and

$$\mathrm{Var}(\boldsymbol{\eta}) = \mathbf{A}^T\mathbf{Z}\mathbf{A} \ ,$$

$$\mathbf{Z} = \mathrm{Var}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) = (\mathbf{A}^T\mathbf{V}^{-1}\mathbf{A})^{-1} \ ,$$

and

$$\mathbf{V} = \sigma_\delta^2\mathbf{B}\mathbf{B}^T + \sigma_\delta^2\mathbf{I} \ ,$$

where **I** denotes the identity matrix. Thus, we have derived the diametral strain model with both the aleatory and epistemic components as required for the input into the CCP uncertainty computation. This model is given by Eqs. (18) and (21); that is,

$$\boldsymbol{s} = \boldsymbol{s}^0 + \mathbf{B}\boldsymbol{\delta}$$

and

$$\hat{S} = \boldsymbol{s}^0 + \boldsymbol{\eta} \ .$$

These two equations are of the form Eqs. (1) and (2).

III.D. Development of the CCP Statistical Error Model

Using the Monte Carlo method discussed in Sec. III.B and the statistical error models of the input variables in Sec. III.C, estimates of $\boldsymbol{\varepsilon}_{ccp}$ and ϑ_{ccp} are readily obtained for further statistical analysis. The development of empirical models, which clearly distinguish between the aleatory and epistemic variables and preserve the more complex structures of the errors, are desirable for accurate NOP trip setpoint solutions. Examining the results of ϑ_{ccp}, we observe a finer error structure between different channels in the core for the aleatory errors.

These channels are IZ and OZ channels, as shown in Fig. 1. For the aleatory variable ϑ_{ccp} (a similar argument holds for $\boldsymbol{\varepsilon}_{ccp}$), let

$$\vartheta_{ccp} = (\vartheta_p^{IZ}, \vartheta_q^{OZ}) ,$$

where ϑ_p^{IZ} with $p = 1, 2, \ldots, P$ are all the channels in the IZ region and ϑ_q^{OZ} with $q = 1, 2, \ldots, Q$ are all the channels in the OZ region.

A five-parameter CCP statistical error model has been proposed that captures the observed phenomenon as follows:

$$\vartheta_p^{IZ} = \Phi_o + \Phi_o^{IZ} + \Phi_p^{IZ} \tag{22}$$

and

$$\vartheta_q^{OZ} = \Phi_o + \Phi_o^{OZ} + \Phi_q^{OZ} , \tag{23}$$

where

Φ_o = variation common to both IZ- and OZ-region channels

Φ_o^{IZ} = variation common to all IZ-region channels

Φ_o^{OZ} = variation common to all OZ-region channels

Φ_p^{IZ} = variation unique to IZ-region channel p

Φ_q^{OZ} = variation unique to OZ-region channel q.

Based on available data, the results indicate that the five parameters Φ_o, Φ_o^{IZ}, Φ_o^{OZ}, Φ_p^{IZ}, and Φ_q^{OZ} are well represented as normal and independently distributed random

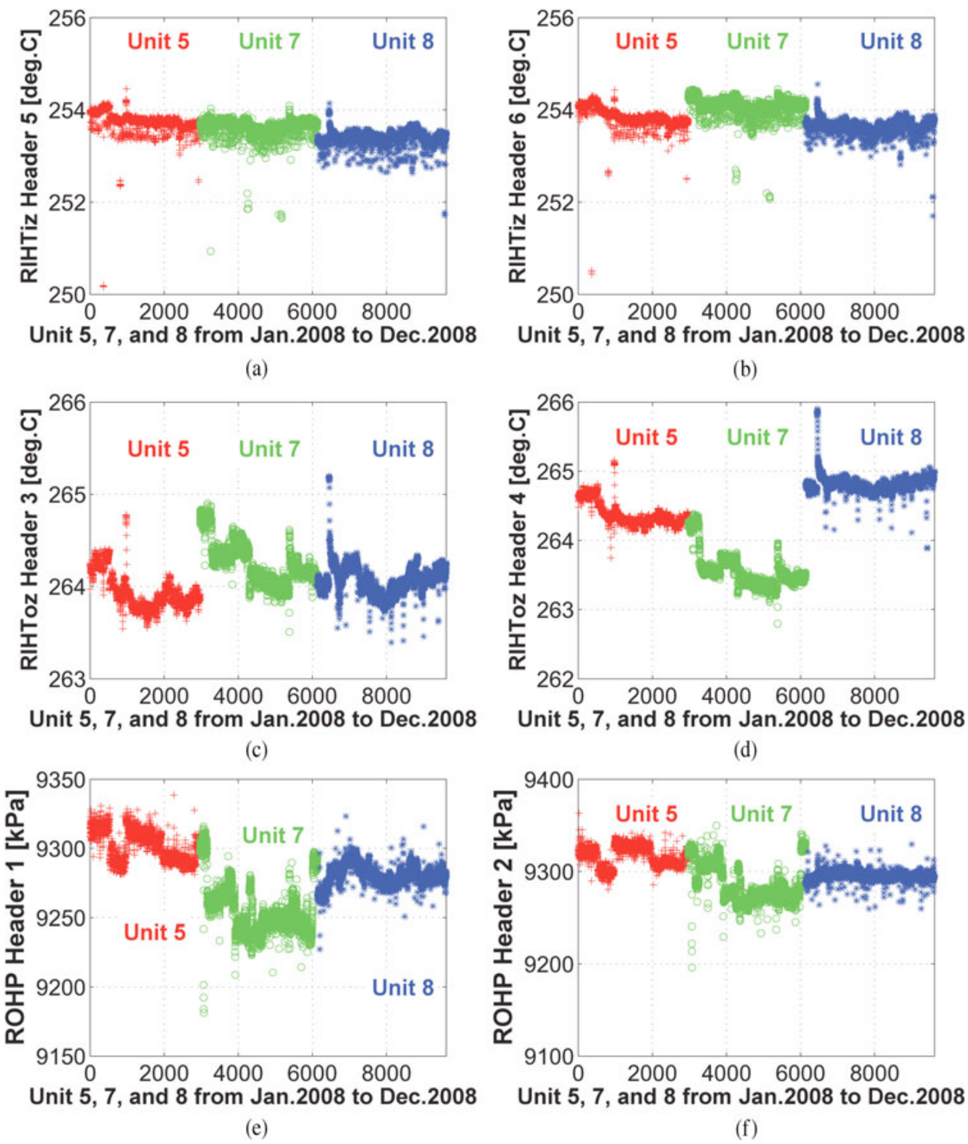

Fig. 2. Time series of raw operational data. (a) through (d) Reactor inlet header temperatures (RIHT) (IZ and OZ). (e) and (f) Reactor outlet header pressure (ROHP).

variables each with zero mean and standard deviations σ_o^2, σ_{oIZ}^2, σ_{oOZ}^2, σ_{IZ}^2, and σ_{OZ}^2, respectively.

Thus, the variance of the CCP aleatory variable for each IZ channel p is given as

$$\mathrm{Var}(\vartheta_p^{IZ}) = \sigma_o^2 + \sigma_{oIZ}^2 + \sigma_{IZ}^2 \ . \tag{24}$$

Similarly, for the OZ-region channel q,

$$\mathrm{Var}(\vartheta_q^{OZ}) = \sigma_o^2 + \sigma_{oOZ}^2 + \sigma_{OZ}^2 \ . \tag{25}$$

As indicated by Eq. (23), the aleatory error ϑ_{ccp} for the IZ channel can be described by a random variable that is common to both IZ- and OZ-region channels, a random variable common to all IZ channels, and a random variable that is unique to the IZ channel p itself.

The covariance of the CCP aleatory variable for each pair of IZ-region channels p_1 and p_2 is given by

$$\mathrm{Cov}(\vartheta_{p_1}^{IZ}, \vartheta_{p_2}^{IZ}) = \sigma_o^2 + \sigma_{oIZ}^2 \text{ for } p_1 \neq p_2 \ . \tag{26}$$

Likewise, a similar decomposition holds for the OZ channels from Eq. (23) with

$$\mathrm{Cov}(\vartheta_{q_1}^{OZ}, \vartheta_{q_2}^{OZ}) = \sigma_o^2 + \sigma_{oOZ}^2 \ , \tag{27}$$

where $q_1 \neq q_2$.

Finally, the covariance of the CCP aleatory variable for each IZ-region channel p with each OZ-region channel q is

$$\mathrm{Cov}(\vartheta_p^{IZ}, \vartheta_q^{OZ}) = \sigma_o^2 \ . \tag{28}$$

Using a method of moments, the five unknowns σ_o^2, σ_{oIZ}^2, σ_{oOZ}^2, σ_{IZ}^2, and σ_{OZ}^2 are estimated using Eqs. (24) through (28). The solutions to the five-parameter CCP error model have been shown to give nonnegative estimates and model the data very well. This is discussed further in Sec. IV.

IV. CCP UNCERTAINTY ANALYSIS RESULTS

Estimates of ϑ_{ccp} and $\boldsymbol{\varepsilon}_{ccp}$ are readily available using the Monte Carlo method and statistical error models associated with each input variable in the CCP BEPU analysis. A unique feature of this approach is that the proposed Monte Carlo method provides a means to accurately capture the statistical dependencies in the system inputs and responses when actual operational data are available (see Fig. 2). The operational data define the initial boundary conditions in the calculation of CCPs and are used in place of Monte Carlo simulations of these input vari-

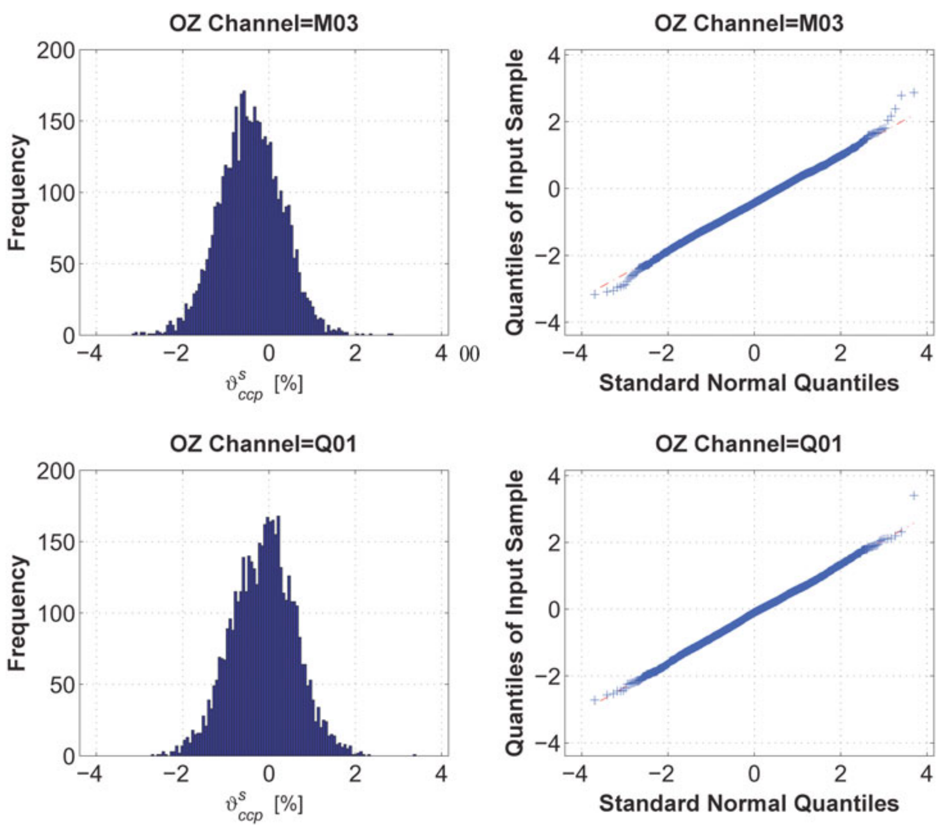

Fig. 3. Monte Carlo analysis results. Histogram and qq-plots of the aleatory error for channels in the OZ (i.e., channels M03 and Q01).

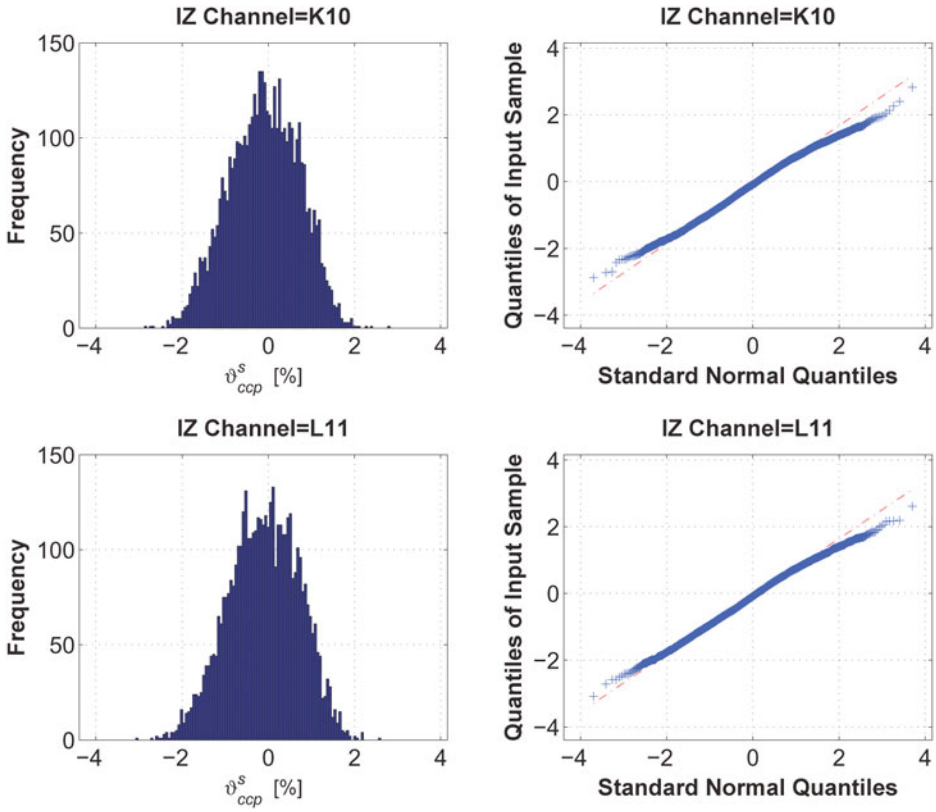

Fig. 4. Monte Carlo analysis results. Histogram and qq-plots of the aleatory error for channels in the IZ (i.e., channels K10 and L11).

ables. This approach accurately reflects the intricate IZ and OZ design of the HTS (see Fig. 1); that is, uncertainties specific to each reactor header are reflected in the response variable (i.e., CCP). This approach eliminates the need to provide accurate estimates of the covariance matrix to describe the multivariate joint probability distributions for these variables that define the initial boundary conditions of the system.

Furthermore, an evaluation of the characteristics of ϑ_{ccp} and $\boldsymbol{\varepsilon}_{ccp}$ using tests for normality and independence is possible using results from the Monte Carlo analysis. As an example,[c] plots of histograms and qq-plots for ϑ_{ccp} associated with typical channels in the IZ and OZ are provided in Figs. 3 and 4, respectively. The results of the qq-plots indicate that ϑ_{ccp} is well represented by normal distributions and support a normal assumption for modeling ϑ_{ccp} for all channels in the core. Furthermore, statistics such as the mean error and standard deviations for ϑ_{ccp} are computable and illustrated in Fig. 5.

Plots of the correlation coefficients are provided in Fig. 6 for the CCP aleatory variable. These plots are used to evaluate the potential correlation structure in ϑ_{ccp}. The correlation results suggest that ϑ_{ccp} cannot be assumed to be independent but exhibit a correlation structure consistent with the specific flow distribution in the core design (i.e., OZ channels and IZ channels). These results indicate that a finer statistical error structure may exist in ϑ_{ccp} and warrant further investigation.

Using the error modeling methodology discussed in Sec. III.C, we estimate the coefficients of the five-parameter CCP error model and use them to describe variations that are either common or unique to the IZ- and OZ-region channels. The randomness in each channel is simulated (i.e., using Monte Carlo) based on the results of the five-parameter model. The correlation coefficients are then computed, and the results are compared against the actual raw data to test the adequacy of the five-parameter model. These results are shown in Fig. 6 and demonstrate that the proposed five-parameter model captures the complex error structure observed in the data very well.

V. SUMMARY

This paper has presented a CCP BEPU analysis using a Monte Carlo approach. The CCP parameter is used as input into the NOP trip setpoint calculations. A key aspect

[c]Note that the same arguments and results hold for ε_{ccp}.

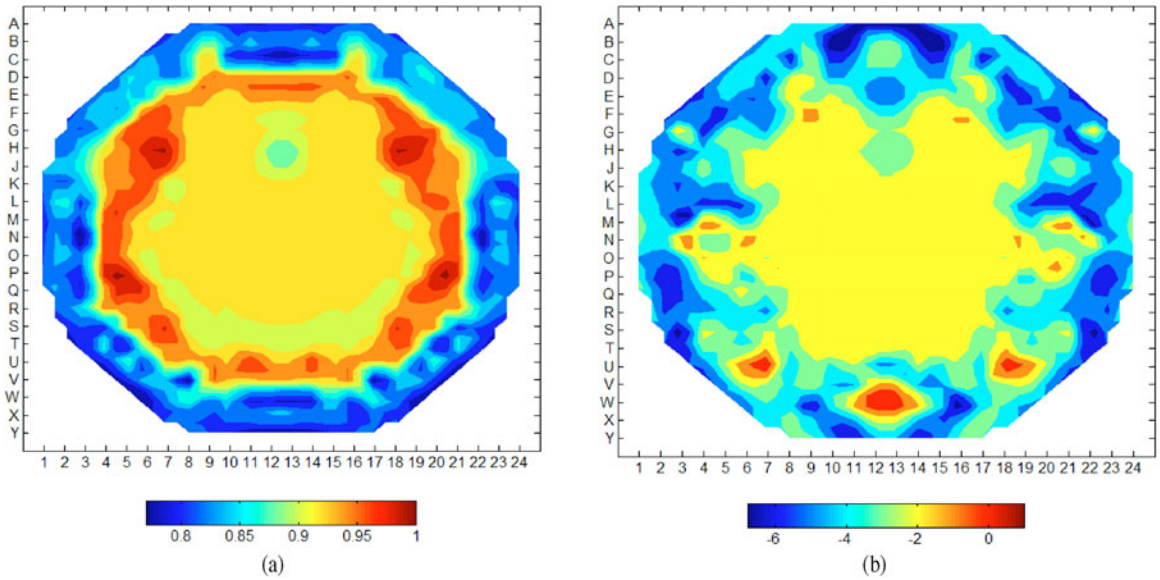

Fig. 5. (a) Standard deviations of ϑ_{ccp}^s for each channel j. (b) Mean error of ϑ_{ccp}^s for each channel j. Values are normalized by the maximum variance of ϑ_{ccp}^s.

of the error modeling is the separation of aleatory and epistemic errors. A five-parameter CCP error model has been proposed to describe variations that are either common or unique to the IZ- and OZ-region channels. This proposed error model has been found to fit the data very well and facilitates the input of what would otherwise be a rather complex statistical structure into the NOP trip setpoint computation.

The proposed Monte Carlo method for error analysis provides improvements in the evaluation of the NOP trip coverage over the traditional methods that assume parametric models that may not accurately represent the statistical error structure.

APPENDIX

SOLUTION TO THE PT STRAIN ERROR MODEL

As discussed in Sec. III.C, the functional form of the PT strain error model is described as follows:

$$S_{ij} = a + b_i \psi_{ij} + c_i \omega_{ij} + \varepsilon_{ij} \ , \tag{A.1}$$

where $\varepsilon_{ij} = \delta_j + \xi_{ij}$. The MLE method is used to solve for the model coefficients $\beta = [a, b_i, c_i]^T$ with the covariance, $V = I_J \otimes (\sigma_\xi^2 I_M + \sigma_\delta^2 u_M u_M^T)$, where u_M is a vector of ones of length M. As before, let S be the vector of length N of all PT strain measurements and $X = [u_N, \Psi, \Omega]$, where u_N is a vector of ones of length N and Ψ and Ω are as defined before. The objective function for the solution of the MLE utilizes the maximization of the likelihood function:

$$L(\beta, \sigma_\delta^2, \sigma_\xi^2) = (2\pi)^{-1/2N} |V|^{-1/2} e^{(-1/2\varepsilon^T V^{-1} \varepsilon)} \ . \tag{A.2}$$

The maximization of the likelihood function L is a nonlinear process given the presence of the nondiagonal covariance matrix V. Thus, the MLE estimates of β, σ_δ^2, and σ_ξ^2 from Eq. (A.2) are obtained by solving the following set of (nonlinear) equations:

$$tr\{V^{-1}\} = (S - X\beta)^T V^{-1} V^{-1} (S - X\beta) \ , \tag{A.3}$$

$$tr\{V^{-1} E_N\} = (S - X\beta)^T V^{-1} E_N V^{-1} (S - X\beta) \ , \tag{A.4}$$

and

$$(X^T V^{-1} X)\beta = X^T V^{-1} S \ , \tag{A.5}$$

where

$$E_N = I_J \otimes E_M$$

$$E_M = u_M u_M^T$$

$tr\{A\}$ = trace of the matrix.

The formula for derivatives of a matrix A with respect to a parameter ϕ are

$$\frac{\partial \ln|A|}{\partial \phi} = tr\left\{A^{-1} \frac{\partial A}{\partial \phi}\right\}$$

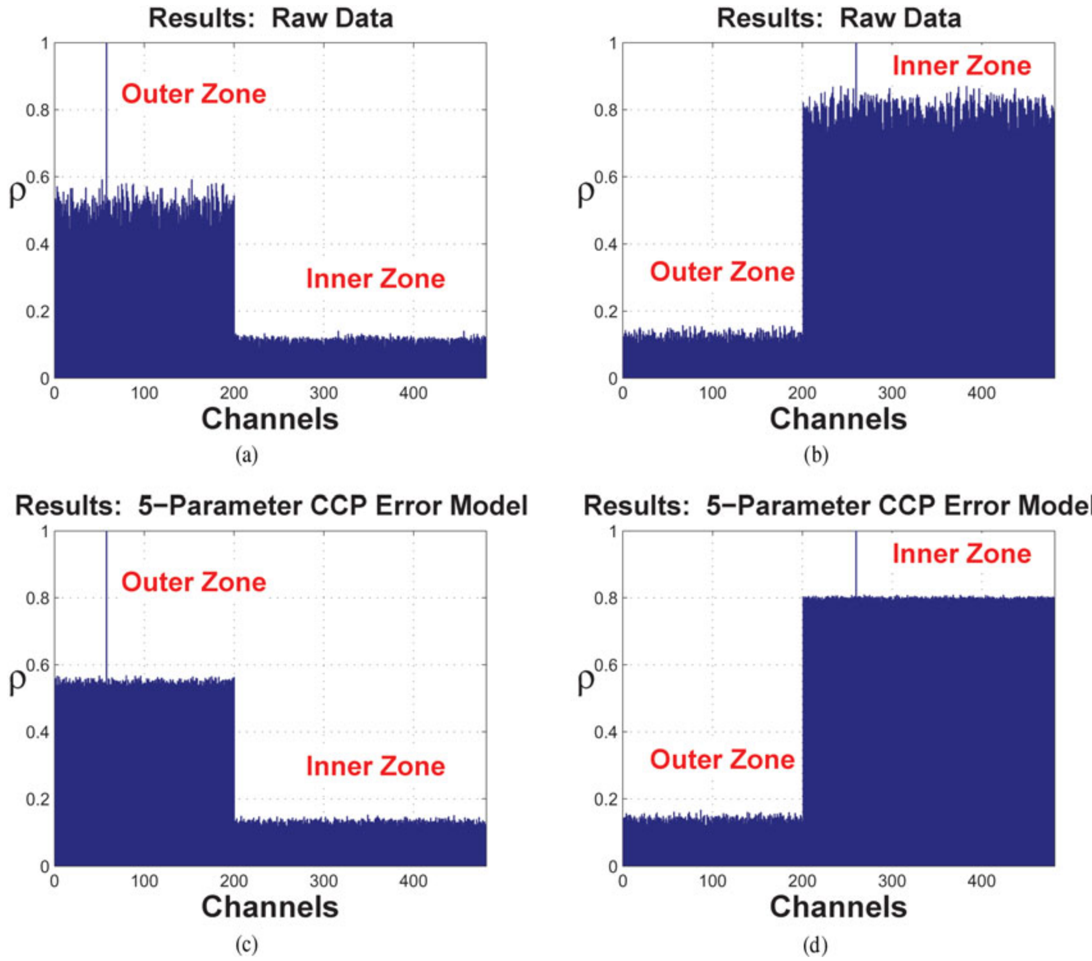

Fig. 6. (a) and (b) Plots of correlation coefficients based on the actual raw data for each channel. (c) and (d) Correlation coefficients based on simulations from the results of the five-parameter CCP error model.

and

$$\frac{\partial A^{-1}}{\partial \phi} = -A^{-1}\frac{\partial A}{\partial \phi}A^{-1} \quad . \tag{A.6}$$

Utilizing the Sherman-Morrison formula for the covariance matrix results in the following expression for its inverse:

$$V^{-1} = \frac{1}{\sigma_\xi^2} I_J \otimes \left[I_M - \frac{1}{M+\kappa} u_M u_M^T \right] \ , \tag{A.7}$$

where $\kappa = \sigma_\xi^2/\sigma_\delta^2$. Using the result from Eq. (A.7) and $E_N^2 = E_N M$ gives

$$V^{-2} = \frac{1}{\sigma_\xi^4}\left(I_N + \frac{(-M-2\kappa)}{(M+\kappa)^2} E_N \right) \quad . \tag{A.8}$$

The trace of the inverse matrix $tr(V^{-1})$ can be evaluated using the result from Eq. (A.7), and using the following properties $tr(I_J \otimes u_M u_M^T) = N$ and $tr(I_N) = N$. This gives Eq. (A.9):

$$tr(V^{-1}) = \frac{N}{\sigma_\xi^2}\left(1 - \frac{1}{M+\kappa} \right) \quad . \tag{A.9}$$

The expression for $tr(V^{-1}E_N)$ reduces to the following form:

$$tr\left\{ \frac{1}{\sigma_\xi^2}\left[I_J \otimes \left(I_M - \frac{u_M u_M^T}{M+\kappa} \right) \right] [I_J \otimes (u_M u_M^T)] \right\} \quad . \tag{A.10}$$

Realizing that $tr(I_J \otimes u_M u_M^T) = N$ and $tr(I_N) = N$, we simplify Eq. (A.10) to

$$tr(V^{-1}E_N) = \frac{N}{\sigma_\delta^2(M+\kappa)} \quad . \tag{A.11}$$

The above system of nonlinear equations for Eq. (A.3) and for Eq. (A.4) leads to Eqs. (A.12) and (A.13), respectively:

$$\frac{N}{\sigma_\xi^2}\left[1-\frac{1}{M+\kappa}\right]=\varepsilon^T\frac{1}{\sigma_\xi^4}\left(I_N+\frac{(-M-2\kappa)}{(M+\kappa)^2}E_N\right)\varepsilon \tag{A.12}$$

and

$$\frac{N}{\sigma_\delta^2(M+\kappa)}=\varepsilon^T\left(\frac{E_N}{\sigma_\delta^4(\kappa+M)^2}\right)\varepsilon \ . \tag{A.13}$$

Thus, the (biased) estimates of σ_δ^2 and σ_ξ^2 and the estimates of the model coefficients β are given as follows:

$$\hat{\beta}=(X^TV^{-1}X)^{-1}X^TV^{-1}S \ , \tag{A.14}$$

$$\hat{\sigma}_\delta^2=\frac{1}{N(M-1)}\hat{\varepsilon}^T(E_N-I_N)\hat{\varepsilon} \ , \tag{A.15}$$

and

$$\hat{\sigma}_\xi^2=\frac{1}{N(M-1)}\hat{\varepsilon}^T(MI_N-E_N)\hat{\varepsilon} \ , \tag{A.16}$$

where $\hat{\varepsilon}=S-X\beta$.

A.I. UNBIASED ESTIMATES FOR THE RANDOM EFFECTS MODEL

The estimated variances, being MLE, are typically biased, and thus, the solution based on an unbiased estimate can be found using expectation as follows:

$$E(\hat{\sigma}_\xi^2)=\frac{1}{N(M-1)}E(\hat{\varepsilon}^T(MI_N-E_N)\hat{\varepsilon})$$

$$=\frac{1}{N(M-1)}tr[E(\hat{\varepsilon}^T(MI_N-E_N)\hat{\varepsilon})] \ , \tag{A.17}$$

where $\hat{\varepsilon}=S-X\hat{\beta}=(I_N-X(X^TV^{-1}X)^{-1}X^TV^{-1})\varepsilon$. Equation (A.17) can be simplified using $E(\hat{\varepsilon}\hat{\varepsilon}^T)=(V-XZX^T)$ and $Z=(X^TV^{-1}X)^{-1}$ as follows:

$$E(\hat{\sigma}_\xi^2)=\frac{1}{N(M-1)}tr[(MI_N-E_N)(V-XZX^T)] \ , \tag{A.18}$$

where

$$V=I_J\otimes(\sigma_\xi^2I_M+\sigma_\delta^2u_Mu_M^T) \ ,$$

$$V^{-1}=\frac{1}{\sigma_\xi^2}I_J\otimes\left[I_M-\frac{1}{M+\kappa}u_Mu_M^T\right] \ ,$$

and

$$\kappa=\sigma_\xi^2/\sigma_\delta^2 \ .$$

Expanding the terms within the trace function gives

$$tr[(MI_N-E_N)(V-XZX^T)]=\sigma_\xi^2\left(N(M-1)-tr\left[(MI_N-E_N)X\left(X^T\left[I_N-\frac{1}{\kappa+M}E_N\right]X\right)^{-1}X^T\right]\right) \ .$$

Thus, the unbiased estimate of σ_ξ^2 is

$$\hat{\hat{\sigma}}_\xi^2=\frac{N(M-1)\hat{\sigma}_\xi^2}{N(M-1)-tr\left[(MI_N-E_N)X\left(X^T\left[I_N-\frac{1}{\kappa+M}E_N\right]X\right)^{-1}X^T\right]} \ . \tag{A.19}$$

An unbiased estimate of the variance σ_δ^2 can similarly be determined:

$$E(\hat{\sigma}_\delta^2)=\frac{1}{N(M-1)}tr[(E_N-I_N)E(\hat{\varepsilon}\hat{\varepsilon}^T)] \ . \tag{A.20}$$

Using $V=I_J\otimes(\sigma_\xi^2I_M+\sigma_\delta^2u_Mu_M^T)$, we can expand (A.20) as follows:

$$E(\hat{\sigma}_\delta^2)=\frac{1}{N(M-1)}tr\left[\sigma_\xi^2(E_N-I_N)+(M-1)\sigma_\delta^2E_N-\sigma_\xi^2(E_N-I_N)X\left(X^T\left[I_N-\frac{1}{\kappa+M}E_N\right]X\right)^{-1}X^T\right] \ .$$

Using $tr(E_N)=N$ and $tr(I_N)=N$, we derive the unbiased estimate of $\hat{\sigma}_\delta^2$ as follows:

$$\hat{\hat{\sigma}}_\delta^2=\frac{N(M-1)\hat{\sigma}_\delta^2}{N(M-1)-\kappa\left(tr\left[(E_N-I_N)X\left(X^T\left[I_N-\frac{1}{\kappa+M}E_N\right]X\right)^{-1}X^T\right]\right)} \ . \tag{A.21}$$

The unbiased estimates of σ_δ^2 and σ_ξ^2 are used to evaluate the revised estimates of $\hat{\beta}$ given in Eq. (A.14).

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