Duration of Examination: 3 hours

M2Z03: SAMPLE FINAL EXAM C

1. The general solution of the differential equation $y' = \dfrac{6\,x^2}{2\,y + \cos y}$ is

(A) $y^2 + \sin y = 2\,x^3 + c$

(B) $2\,y^2 + \cos y = 6\,x^3 + c$

(C) $2\,y^2 + \sin y = 6\,x^3 + c$

(D) $y^2 + \cos y = x^3 + c$

(E) $y^2 + \sin y = x^2 + c$

2. The solution of the initial value problem $(1 + x^2)y' + 2x\,y = \cos x,\ y(0) = 1$ is

(A) $y = \dfrac{1 + x^2}{\sin(x)}$

(B) $y = \dfrac{x^2 + 2}{2 + \sin(x)}$

(C) $y = \dfrac{x^2 + 1}{1 + \cos(x)}$

(D) $y = \dfrac{1 + \sin(x)}{x^2 + 1}$

(E) $y = \dfrac{2 - \cos(x)}{x^2 + 1}$

3. Which of the following homogeneous linear differential equations has $y(x) = x\,\sin(\sqrt{2}\,x)$ as a solution?

(A) $y'' + 2\,y = 0$

(B) $y'' + 2\,y' + 2\,y = 0$

(C) $y^{(4)} + 4\,y'' + 4\,y = 0$

(D) $y^{(4)} + 2\,y'' + 2\,y = 0$

(E) None of the above

4. Which of the following is a fundamental set of solutions for the homogeneous differential equation $y'' - 6\,y' + 9\,y = 0$?

(A) $\{e^{3x}, e^{3x}\}$

(B) $\{e^{-3x}, e^{3x}\}$

(C) $\{e^{3x}, x\,e^{3x}\}$

(D) $\{e^{-3x}, x\,e^{-3x}\}$

(E) $\{e^{x/3}, e^{-x/3}\}$

5. The Wronskian determinant $W(x)$ associated with the set of functions $\{x, e^x, x + e^x\}$ is

(A) e^x

(B) e^{2x}

(C) xe^{2x}

(D) 0

(E) 1

6. The homogeneous equation $y'' + 16y = 0$ has for general solution

$$y(x) = c_1 \cos(4x) + c_2 \sin(4x)$$

(you need not check that). The boundary–value problem $y''+16\,y = 0$, $y(0) = 0$, $y(\pi) = 1$ has:

(A) the unique solution $y = 1 - \cos(4x)$

(B) no solution

(C) the unique solution $y = \sin(4x)$

(D) two solutions: $y = \cos(4x)$ and $y = \sin(4x)$

(E) infinitely many solutions: $y = C\sin(4x)$, where C is any number

7. The homogeneous differential equation $y^{(4)} + y^{(3)} - 2\,y'' = 0$ has for general solution

$$y(x) = c_1 + c_2\,x + c_3\,e^{-2\,x} + c_4\,e^{x}$$

(you need not check that). For this equation, the initial value problem $y(0) = 1$, $y'(0) = -1$, $y''(0) = 4$, $y^{(3)}(0) = -8$ has solution

(A) $y(x) = 0$

(B) $y(x) = x + e^{-x}$

(C) $y(x) = 1 + e^{-2\,x}$

(D) $y(x) = x + e^{-2\,x}$

(E) $y(x) = 1 + e^{-x}$

8. The solution of the initial value problem $4\,y'' - y = x\,e^{\frac{x}{2}}$ subject to initial condition $y(0) = 1$, $y'(0) = 0$ is

(A) $y(x) = \frac{3}{4}e^{\frac{x}{2}} + \frac{1}{4}e^{-\frac{x}{2}} + \frac{1}{8}\,x^2 e^{\frac{x}{2}} - \frac{1}{4}\,xe^{\frac{x}{2}}$

(B) $y(x) = e^{\frac{x}{2}} + e^{-\frac{x}{2}} + x^2 e^{\frac{x}{2}} - xe^{\frac{x}{2}}$

(C) $y(x) = \frac{3}{4}e^{-\frac{x}{2}} + \frac{1}{4}e^{\frac{x}{2}} + \frac{1}{8}\,x^2 e^{-\frac{x}{2}} - \frac{1}{4}\,xe^{-\frac{x}{2}}$

(D) $y(x) = \frac{7}{8}$

(E) None of the above

9. If the method of undetermined coefficients is used to solve the linear differential equation $y'' + 9\,y = \sin(3\,x)$, then the form of a particular solution is

(A) $y_p(x) = A\sin(3\,x) + B\cos(3\,x)$

(B) $y_p(x) = A\,x\sin(3\,x) + B\,x\cos(3\,x)$

(C) $y_p(x) = A\sin(3\,x)$

(D) $y_p(x) = A\,x^2\sin(3\,x) + B\,x^2\cos(3\,x)$

(E) $y_p(x) = A\,x^3\sin(3\,x) + B\,x^3\cos(3\,x)$

10. The general solution of the linear differential equation $x^2\,y'' - 6\,x\,y' - 8\,y = x^2$, for which the homogeneous part has the solutions $y_1(x) = x^8$ and $y_2(x) = \frac{1}{x}$, is given by

(A) $y(x) = c_1\,x^8 + \dfrac{c_2}{x} - \dfrac{1}{48\,x^6}$

(B) $y(x) = c_1\,x^8 + \dfrac{c_2}{x} - \dfrac{1}{27\,x^3}$

(C) $y(x) = c_1\,x^8 + \dfrac{c_2}{x} - \dfrac{1}{48\,x^2}$

(D) $y(x) = c_1\,x^8 + \dfrac{c_2}{x} - \dfrac{1}{12}\,x^4$

(E) $y(x) = c_1\,x^8 + \dfrac{c_2}{x} - \dfrac{1}{18}\,x^2$

11. Consider the boundary–value problem $y'' + \lambda\,y = 0$, $y'(0) = 0$, $y'(\pi) = 0$. The smallest eigenvalue λ for this problem is

(A) 1

(B) $\frac{1}{2}$

(C) 0

(D) $\frac{1}{4}$

(E) π

12. Suppose that A is a 2×2 real matrix and that one of its eigenvalue is $\lambda = 2 + i$ with corresponding eigenvector $v = \begin{bmatrix} 2 \\ i \end{bmatrix}$. Then, the general solution of the 1st order system $\mathbf{X}' = A\mathbf{X}$ is given by

(A) $\mathbf{X}(t) = c_1 \begin{bmatrix} e^t \cos(2t) \\ -2e^t \sin(2t) \end{bmatrix} + c_2 \begin{bmatrix} e^t \sin(2t) \\ 2e^t \cos(2t) \end{bmatrix}$

(B) $\mathbf{X}(t) = c_1 \begin{bmatrix} 2e^{2t} \cos t \\ -e^{2t} \sin t \end{bmatrix} + c_2 \begin{bmatrix} 2e^{2t} \sin t \\ e^{2t} \cos t \end{bmatrix}$

(C) $\mathbf{X}(t) = c_1 \begin{bmatrix} 2e^{2t} \sin t \\ -e^{2t} \cos t \end{bmatrix} + c_2 \begin{bmatrix} -e^{2t} \sin t \\ 2e^{2t} \cos t \end{bmatrix}$

(D) $\mathbf{X}(t) = c_1 \begin{bmatrix} 2e^t \cos t \\ -e^t \sin t \end{bmatrix} + c_2 \begin{bmatrix} e^{2t} \sin t \\ 2e^{2t} \cos t \end{bmatrix}$

(E) $\mathbf{X}(t) = c_1 \begin{bmatrix} 2e^{2t} \sin t \\ -e^{2t} \sin t \end{bmatrix} + c_2 \begin{bmatrix} 2e^{2t} \cos t \\ e^{2t} \cos t \end{bmatrix}$

13. Suppose that A is a 3×3 real matrix with characteristic polynomial

$$P(\lambda) = \det(A - \lambda I) = -\lambda^3 + 2\lambda + 4.$$

Find A if

$$A^3 = \begin{bmatrix} 8 & 4 & -2 \\ 0 & 2 & 2 \\ 0 & -2 & 2 \end{bmatrix}.$$

(A) $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -2 \\ -2 & -2 & 2 \end{bmatrix}$

(B) $A = \begin{bmatrix} 2 & 1 & -2 \\ 1 & 2 & -1 \\ 0 & -2 & 1 \end{bmatrix}$

(C) $A = \begin{bmatrix} 0 & 1 & -2 \\ 2 & 1 & 1 \\ 1 & -2 & 4 \end{bmatrix}$

(D) $A = \begin{bmatrix} 0 & 2 & -1 \\ 2 & 1 & 2 \\ 4 & -1 & 2 \end{bmatrix}$

(E) $A = \begin{bmatrix} 2 & 2 & -1 \\ 0 & -1 & 1 \\ 0 & -1 & -1 \end{bmatrix}$

14. The symmetric matrix

$$A = \begin{bmatrix} 8 & -2 & -2 \\ -2 & 11 & -1 \\ -2 & -1 & 11 \end{bmatrix}.$$

has characteristic polynomial $P(\lambda) = -(\lambda - 6)(\lambda - 12)^2$ and $v_1 = \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}$ is an eigenvector associated with the eigenvalue $\lambda = 12$. Another eigenvector associated with the eigenvalue $\lambda = 12$ and orthogonal to v_1 is given by

(A) $v_2 = \begin{bmatrix} -4 \\ 1 \\ 7 \end{bmatrix}$

(B) $v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

(C) $v_2 = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$

(D) $v_2 = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$

(E) $v_2 = \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}$

15. Let

$$A = \begin{bmatrix} 2 & a \\ 3 & -1 \end{bmatrix}$$

Find the value of a for which $\lambda = 5$ is an eigenvalue of A.

(A) $a = 2$

(B) $a = -5$

(C) $a = -3$

(D) $a = 6$

(E) $a = 7$

16. Find the value of a for which the vector $v = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ is an eigenvector of the matrix

$$A = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & a \\ a & 2 & 0 \end{bmatrix}.$$

(A) $a = 1$

(B) $a = -1$

(C) $a = -3$

(D) $a = 2$

(E) $a = 0$

17. The eigenvalues λ of the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 2 \\ 13 & 1 & 0 \end{bmatrix}$$

are given by:

(A) $\lambda = 0, 2, 3$

(B) $\lambda = -2, 1, 4$

(C) $\lambda = -3, 2, 5$

(D) $\lambda = -1, 1, 2$

(E) $\lambda = -13, 2, 3$

18. A 3×3 matrix A has eigenvalues $\lambda_1 = -2$, $\lambda_2 = 1$ and $\lambda_3 = 4$ with corresponding eigenvectors

$$v_1 = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix},$$

respectively. Then, a matrix P satisfying

$$P^{-1}AP = \begin{bmatrix} 4 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

is given by:

(A) $P = \begin{bmatrix} -1 & 0 & 2 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

(B) $P = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

(C) $P = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

(D) $P = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 2 & 2 \end{bmatrix}$

(E) $P = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix}$

19. The 3rd order linear differential equation $x'''(t) - 2\,x''(t) + 3\,x(t) = 0$ can be written as a 1st order linear system of the form $\mathbf{X}'(t) = A\,\mathbf{X}(t)$ where $\mathbf{X}(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}$, with $y(t) = x'(t)$, $z(t) = x''(t)$ and A being the matrix

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & -2 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 2 & 0 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & 0 & 2 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 3 & -2 & 1 \end{bmatrix}$

(E) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ -3 & 2 & 0 \end{bmatrix}$

20. The solution of the initial value problem

$$\begin{cases} x'(t) = -x(t), \\ y'(t) = -4\,x(t) + 3\,y(t), \end{cases}$$

with initial conditions $x(0) = 2$, $y(0) = 3$ is given by

(A) $x(t) = e^{-t} + e^{3t},\ y(t) = e^{-t} + 2\,e^{3t}$

(B) $x(t) = 3\,e^{-t} - e^{3t},\ y(t) = 2\,e^{-t} + e^{3t}$

(C) $x(t) = 2\,e^{-t},\ y(t) = 2\,e^{-t} + e^{3t}$

(D) $x(t) = -e^{-t} + 2\,e^{3t},\ y(t) = -e^{-t} + 3\,e^{3t}$

(E) $x(t) = 2\,e^{-t},\ y(t) = -e^{-t} + 4\,e^{3t}$

21. Suppose that A is a real, 3×3 matrix with eigenvalues $\lambda_1 = -1$, $\lambda_2 = 0$ and $\lambda_3 = 2$ and corresponding eigenvectors

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix},$$

respectively. Let $\mathbf{X}(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}$ be the unique solution of the initial value problem $\mathbf{X}'(t) = A\,\mathbf{X}(t)$ with initial condition $\mathbf{X}(0) = \begin{bmatrix} 3 \\ 3 \\ 9 \end{bmatrix}$. Then, the value of the 1st component of $\mathbf{X}(t)$ at $t = 1$ is

(A) $x(1) = 2 + e$

(B) $x(1) = e + 2\,e^{-1}$

(C) $x(1) = 1 + 2\,e^{-1}$

(D) $x(1) = -1 + e^{-1} + 2\,e$

(E) $x(1) = 1 - 2\,e^{-1} - e$

22. Let A be an $n \times n$ real, symmetric matrix. Which of the following statements is **not always** correct?

(A) $A\,B = B\,A$ for any $n \times n$ symmetric matrix B
(B) A is diagonalizable
(C) $A^2 - A$ is symmetric
(D) A^2 is symmetric
(E) A^2 is diagonalizable

23. Let A be an $n \times n$ real matrix. Which of the following does **not** necessarily imply that A is diagonalizable:

(A) A has n distinct eigenvalues.
(B) A is symmetric.
(C) $A = B^2$, where B is diagonalizable.
(D) A is invertible and A^{-1} is diagonalizable.
(E) The 1st order system $\mathbf{X}'(t) = A\,\mathbf{X}(t)$, where $\mathbf{X}(t)$ is a column vector with n components, admits n linearly independent solutions for $-\infty < t < \infty$.

24. The Laplace transform $\mathcal{L}\left\{(3\,t-1)^2\right\}$ is

(A) $\frac{18}{s^3} - \frac{6}{s^2} + \frac{1}{s}$
(B) $\frac{9}{s^2} - \frac{6}{s} + 1$
(C) $9\,s\,e^{-s/3}$
(D) $\frac{(3\,s-1)^2}{s^3}$
(E) $\frac{1}{9}\,s\,e^{s/3}$

25. The Laplace transform $\mathcal{L}\left\{e^{3t-2}\right\}$ is

(A) $\frac{1}{3\,s-5}$
(B) $\frac{3}{s-\frac{2}{3}}$
(C) $\frac{e^2}{s+3}$
(D) $\frac{e^{-2}}{s-3}$
(E) $\frac{1}{(3\,s-2)^3}$

26. The Laplace transform $\mathcal{L}\left\{t\,\sin(2t)\right\}$ is

(A) $\frac{4}{(s^2+4)^2}$
(B) $\frac{4\,s}{(s^2+4)^2}$
(C) $-\frac{4\,s}{(s^2+4)^2}$
(D) $\frac{2\,s}{(s^2+4)^2}$
(E) $-\frac{2\,s^2}{s^2+4}$

27. The inverse Laplace transform $\mathcal{L}^{-1}\left\{\dfrac{1}{s\,(s^2+4)}\right\}$ is

(A) $1-\cos(2t)$

(B) $4\,(1-\cos(2t))$

(C) $\dfrac{1}{4}\,(1-\cos(2t))$

(D) $\dfrac{1}{2\,t}\,\sin(2t)$

(E) $\dfrac{1}{4}\,(1+\cos(2t))$

28. Suppose that $\mathcal{L}\{f(t)\} = \dfrac{s\,e^{-\pi s}}{s^2+4}$. Then, the value $f(3\,\pi)$ is

(A) 0

(B) 1

(C) -1

(D) $\dfrac{1}{2}$

(E) $\dfrac{3\,\pi\,e^{-3\,\pi^2}}{9\,\pi^2+4}$

29. The convolution $(f*g)\,(t)$ of $f(t)=e^{-2t}$ and $g(t)=e^{2t}$ is equal to

(A) 1

(B) t

(C) $\dfrac{1}{2}\,\left(e^{2t}-e^{-2t}\right)$

(D) $\dfrac{1}{4}\,\left(e^{2t}+e^{-2t}\right)$

(E) $\dfrac{1}{4}\,\left(e^{2t}-e^{-2t}\right)$

30. Consider the differential equation

$$y' + 2\,y = \begin{cases} 1, & 0 \le t < 3, \\ 0, & t \ge 0, \end{cases}$$

with initial condition $y(0)=0$. The Laplace transform $\mathcal{L}\{y(t)\}$ of the solution is

(A) $\dfrac{e^{-3s}}{s+2}$

(B) $\dfrac{1-e^{-3s}}{s\,(s+2)}$

(C) $\dfrac{1-e^{-2s}}{s\,(s+3)}$

(D) $\dfrac{e^{-3s}-1}{s\,(s+2)}$

(E) $\dfrac{1+e^{-3s}}{s\,(s+2)}$

31. If $y(t)$ is the solution of the differential equation $y''+\dfrac{1}{4}\,y=\delta(t-\pi)$ with initial conditions $y(0)=0$, $y'(0)=1$, then $y(2\pi)$ is equal to

(A) 2

(B) -2

(C) 0

(D) 4

(E) -4

32. The system of differential equations

$$\begin{cases} x' = 2\,x - y \\ y' = 4\,x - 2\,y \end{cases}$$

with initial conditions $x(0) = 1$ and $y(0) = 1$ has for solution

(A) $x(t) = 2\,e^{2t} - e^{t}$

(B) $x(t) = 1 + 2\,t$

(C) $x(t) = 1 - t$

(D) $x(t) = 1 + t$

(E) $x(t) = e^{2t}$

33. The differential equation $(4 - x^2)\,x^3\,y''(x) + x\,y'(x) - 2\,y(x) = 0$ has all of its singular points classified as

(A) $x = 2$ regular, $x = -2$ regular, $x = 0$ irregular

(B) $x = 4$ regular, $x = 0$ regular

(C) $x = 4$ regular, $x = -4$ regular, $x = 0$ regular

(D) $x = 2$ irregular, $x = -2$ irregular, $x = 0$ regular

(E) $x = 2\,i$ regular, $x = -2\,i$ regular, $x = 0$ irregular

34. The differential equation $9\,x\,y'' + 3\,y' + x\,y = 0$ has a general solution of the form

(A) $y = \sum_{n=0}^{\infty} a_n\,x^n + \sum_{n=0}^{\infty} b_n\,x^{n+\frac{2}{3}}$

(B) $y = \sum_{n=0}^{\infty} a_n\,x^n + \sum_{n=0}^{\infty} b_n\,x^{n-\frac{2}{3}}$

(C) $y = \sum_{n=0}^{\infty} a_n\,x^{n+\frac{1}{3}} + (\ln x)\sum_{n=0}^{\infty} b_n\,x^{n+\frac{1}{3}}$

(D) $y = \sum_{n=0}^{\infty} a_n\,x^{n-\frac{1}{3}} + (\ln x)\sum_{n=0}^{\infty} b_n\,x^{n-\frac{1}{3}}$

(E) $y = \sum_{n=0}^{\infty} a_n\,x^n + \sum_{n=0}^{\infty} b_n\,x^{n-\frac{1}{3}}$

35. The differential equation $y'' + x\,y' + y = 0$ has a power series solution $y = \sum_{n=0}^{\infty} a_n\,x^n$ where the recursion formula for the coefficients a_n is

(A) $a_{n+2} = \dfrac{n-1}{(n+2)\,(n+1)}\,a_n$

(B) $a_{n+1} = -\dfrac{1}{n+2}\,a_n$

(C) $a_{n+2} = -\dfrac{1}{n+2}\,a_n$

(D) $a_{n+2} = -\dfrac{n-1}{(n+2)\,(n+1)}\,a_n$

(E) $a_{n+2} = -\dfrac{1}{n+1}\,a_{n+1} - \dfrac{1}{n+2}\,a_n$

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Some formulas you may use:

$$y_p(x) = -\left[\int \frac{y_2(x)\,f(x)}{W(y_1,y_2)}\,dx\right] y_1(x) + \left[\int \frac{y_1(x)\,f(x)}{W(y_1,y_2)}\,dx\right] y_2(x)$$

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty f(t)\,e^{-st}\,dt.$$

$$\mathcal{L}\{1\} = \frac{1}{s}, \quad \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}, \quad \mathcal{L}\{e^{a\,t}\} = \frac{1}{s-a},$$

$$\mathcal{L}\{\sin(k\,t)\} = \frac{k}{s^2+k^2}, \quad \mathcal{L}\{\cos(k\,t)\} = \frac{s}{s^2+k^2},$$

$$\mathcal{L}\{f'(t)\} = s\,F(s) - f(0), \quad \mathcal{L}\{f''(t)\} = s^2\,F(s) - s\,f(0) - f'(0),$$

$$\mathcal{L}\{f^{(n)}(t)\} = s^n\,F(s) - s^{n-1}\,f(0) - s^{n-2}\,f'(0) - \cdots - s\,f^{(n-2)}(0) - f^{(n-1)}(0),$$

$$\mathcal{L}\{e^{a\,t}\,f(t)\} = F(s-a), \quad \mathcal{L}\{\mathcal{U}(t-a)\,f(t-a)\} = e^{-s\,a}\,F(s),$$

$$\mathcal{L}\{t^n\,f(t)\} = (-1)^n\,\frac{d^n}{ds^n}F(s),$$

$$\mathcal{L}\{f*g\} = \mathcal{L}\left\{\int_0^t f(t-\tau)\,g(\tau)\,d\tau\right\} = F(s)\,G(s),$$

$$\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{F(s)}{s},$$

$$\mathcal{L}\{\delta(t-t_0)\} = e^{-s\,t_0}.$$

$$2\,\sin(A)\,\sin(B) = \cos(A-B) - \cos(A+B),$$

$$2\,\sin(A)\,\cos(B) = \sin(A+B) + \sin(A-B),$$

$$2\,\cos(A)\,\cos(B) = \cos(A+B) + \cos(A-B),$$

$$\sin(A+B) = \sin(A)\,\cos(B) + \cos(A)\,\sin(B),$$

$$\cos(A+B) = \cos(A)\,\cos(B) - \sin(A)\,\sin(B).$$

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