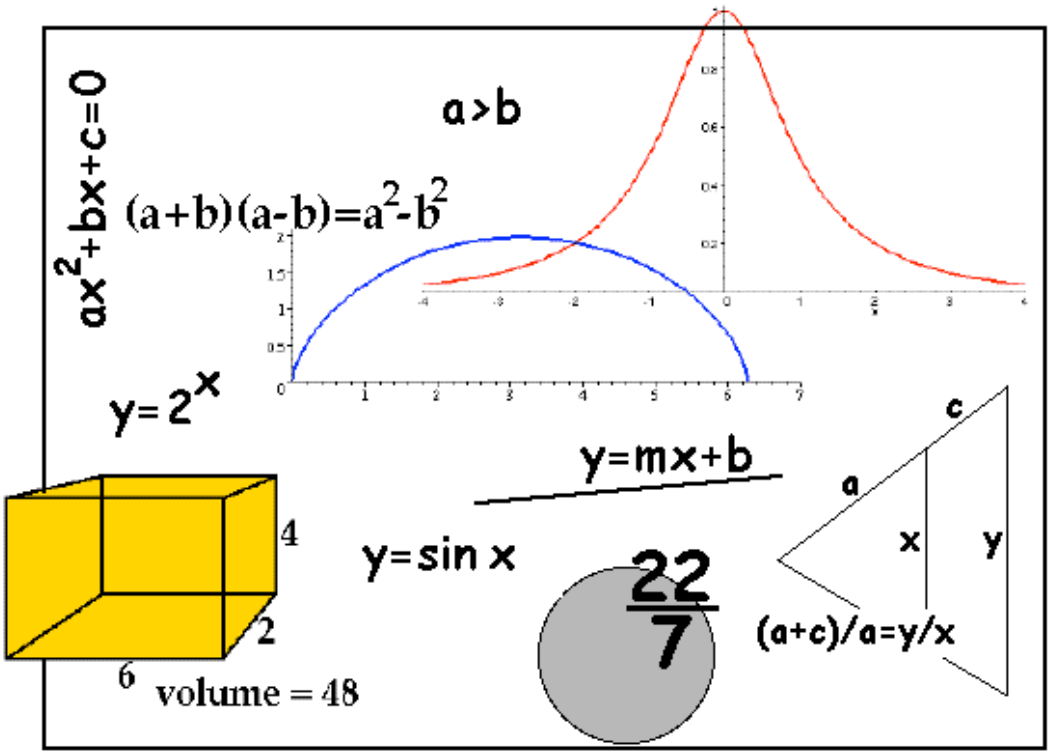
Mathematics Review Manual

With a Brief

First-year Survival Guide

For Students Entering McMaster University



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Also available online at
www.math.mcmaster.ca/lovric/rm.html

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This booklet will help you prepare
for your academic life as a first-year university student,
and, in particular,
for the first-year courses in mathematics
that you will take.

Table of Contents

General Information

Mathematics

You have to know and be proficient in the material from the following four chapters. Very little of it will be reviewed in class.

The material from the three chapters below will be taught and discussed in your first-year calculus course. However, you will have to spend extra time working on these areas, to gain technical proficiency and confidence with the material.

The material from the chapter below will be covered in depth in your first-year calculus course. If you decide to skip something in this manual, then skip this chapter.

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This manual is also available online at (free download)
www.math.mcmaster.ca/lovric/rm.html

What's this manual and survival guide about?

A leap from secondary education to university environment will be, without doubt, one of the most challenging and stressful events in your life. It is a true rite of passage, with all of its anxieties, pains, hopes, frustrations, joys and rewards. You have probably created a mental image of the new environment you will be encountering soon - but it is blurry, lots of fine detail is missing. The better prepared you are, the easier it will be to adjust to new situations, demands and expectations that university life will place on you.

No matter which high school you came from, you have certain strengths and certain weaknesses. There are things that you learnt well in high school, things you know and are comfortable with. But, there are things that you forgot, or you don't know about or have very little experience with. In high school you acquired certain skills, but need to brush up on some others.

This manual will tell you where you are; it will help you identify those areas of mathematics that you are good at, and those areas that you need to learn, review and work on. All you need is a little dedication, a pencil and paper, and about an hour of your (uninterrupted) time per day (say, during the last three weeks of August). Unplug the TV, turn off your cell phone, kick your sibling(s) and/or your parent(s) out of your room, and tell them that you need to work on something really important.

This manual has two parts. The first part is about the things you have thought a lot lately. How is life in university different from high school? What should I expect from my fist-year classes? How is university math different from math in high school? Read, and reflect on the issues raised ... discuss it with your parents, friends, teachers, or older colleagues. Nobody can give you detailed and precise answers to all questions that you have, but at least, you will get a good feeling about the academic side of your first-year university experience.

What will my first-year professors assume that I know about mathematics? The big part of this booklet is dedicated to answering this question. Look at the table of contents to see what areas of mathematics are covered.

As I said, have a pencil and paper handy. I suggest that you start with the first section, and work from there, without skipping sections. Read the material

presented in a section slowly, with understanding; make notes and try to solve exercises as you encounter them (answers to all exercises are in the back of this manual). Even if the material in some section looks easy, do not skip the whole section – select several problems and test your knowledge.

If you realize that you have **problems with certain material**, read the section carefully, twice or three times if needed. Work on problems slowly, making sure that you understand what is going on. If needed, consult your high school textbooks, or go to a local library and find a reference. Ask somebody who knows the stuff to discuss it with you; if you prefer, hire a private math tutor for a few sessions.

I know that doing math is not the coolest thing to do in summer - BUT think a bit about the future. Change from high school to university is a big change; the better prepared you are, the easier it will be for you to adjust successfully to your new life as a university student. Student life is a busy life. It will be quite difficult for you (I did not say impossible!) to find time to do two things: learn new material presented in a lecture and, at the same time, review background material that you are assumed to know and be comfortable with. Not to mention that, without adequate preparation, you will have difficulties following lectures. **Review your math now, while you have lots of free time on your hands!**

One thing is certain: **the more math you do, the easier it gets** - experience helps! **Do as many problems as you can, don't give up because the stuff looks difficult or you feel bored with it.** Little investment of your time now, in summer, will make studying mathematics in the fall a whole lot easier.

Good luck!

See you in September,

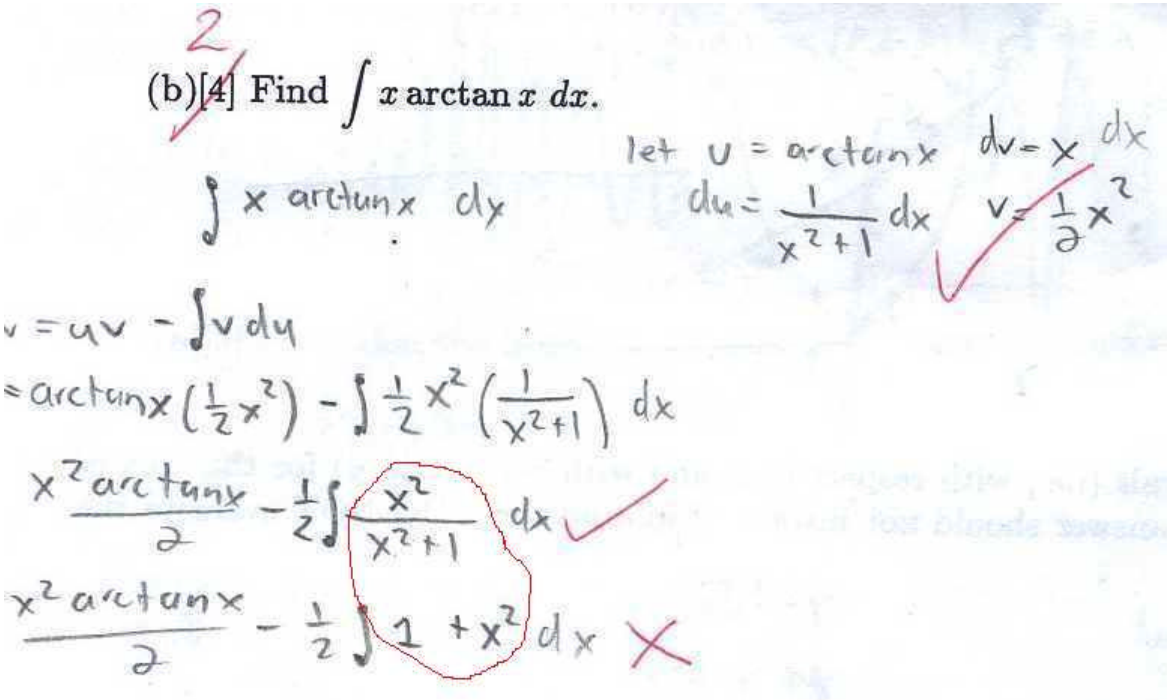
Miroslav Lovric, Associate Professor
First-year instructor
Department of Mathematics and Statistics, McMaster University

Why Background Knowledge Matters

Mathematics is cumulative, **new material builds upon the previously covered (i.e., understood, learnt) material.** It is not possible to truly understand and apply an advanced concept (say, derivatives) without understanding all basic concepts that are used to define it (fractions, limits, graphs, etc.).

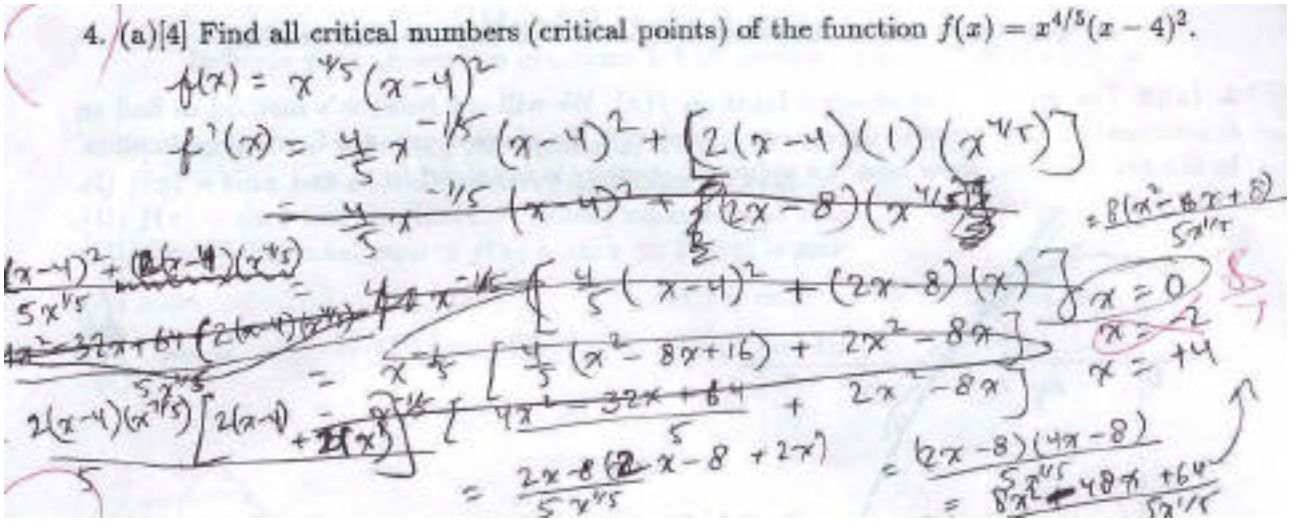
Many times, **the reason why students lose marks on tests in first-year Calculus (and other math courses!) is due to a problem with something elementary**, such as fractions, simplifying, solving equations, or recalling basic properties of exponents, trigonometric functions. etc. Let us look at a few samples of actual test solutions.

In the case below, the student chose the appropriate integration method (which is taught in the first-year Calculus course), but then did not simplify correctly the fraction in the integral (see the last two lines). This error cost the student 50% of the credit for the question.



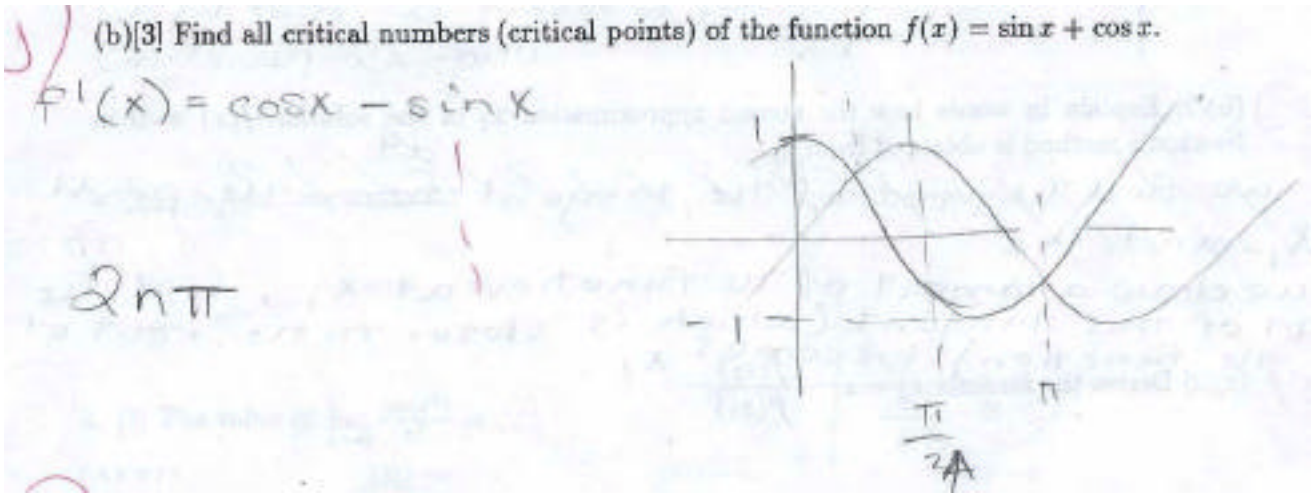
Look how mach effort was put into simplifying the expression for f'(x) below – not to mention **how much valuable time was lost!** Moreover, the student made a

mistake in simplifying and got two (of three) correct values for x. Penalty for this mistake: 25% of the credit for the question. As in the previous case, **note that the credit lost was not due to a new concept learnt in the university Calculus course, but due to errors related to high school material.**



Most-often-heard comment about a test is that **'there was not enough time.' Certainly, if it takes you more than 5 minutes to do this question, you will not have enough time to complete the test.**

In the case below, the student tried to analyze the expression for f'(x) by looking at the graphs of sin x and cos x (excellent idea!). However, the graphs of the two functions are incorrect, and the answer does not make sense. The student lost all credit for the question.



TRANSITION FROM HIGH SCHOOL TO UNIVERSITY

What is new and different in university? Well, almost everything: **new people** (your peers/colleagues, teaching and lab assistants, teachers, administrators, etc.), **new environment, new social contexts, new norms,** and – very important - **new demands and expectations. Think about the issues raised below. How do you plan to deal with it? Read tips and suggestions, and try to devise your own strategies.**

First-year lectures are large – you will find yourself in a huge auditorium, surrounded by 300, 400, or perhaps even more students. Large classes create **intimidating situations**. You listen to a professor lecturing, and hear something that you do not understand. Do you have enough courage to rise your hand and ask the lecturer to clarify the point? Keep in mind that you are not alone – other students feel the same way you do. It's hard to break the ice, but you have to try. Other students will be grateful that you asked the question – you can be sure that lots of them had exactly the same question in mind.

Remember, **learning is your responsibility**. Come to classes regularly, be active, take notes, ask questions. Find a quiet place to study. Use all resources available to you. Discuss material with your colleagues, teaching and lab assistants, and/or professors.

Courses have different requirements and restrictions with regards to **calculators and computer software**. You will find the information about it in the course syllabus that will be given to you (usually) in the first lecture of a course.

The **amount of personal attention** you get from your teachers, compared to high school, is drastically lower. If you have a question, or a problem, you will have to make an effort to talk to your lecturer, or to contact the most appropriate person.

Consider taking courses that help you develop research skills (such as: critical use of electronic resources, logical and critical thinking, library search skills, communication and presentation skills etc.). Have you heard of **inquiry courses**?

Good time management is essential. Do not leave everything for the last moment. Can you complete three assignments in one evening? Or write a major essay and prepare for a test in one weekend? Plan your study time carefully. Eat well, exercise regularly, plan social activities - have a life! **Amount of material covered** in a unit of time increases at least three-fold in university courses, compared to high school. This means that things happen very quickly. If you miss classes and do not study regularly you will get behind in your courses. Trying to catch up is not easy. **For each hour of lecture** plan to spend (at least) three hours studying, reviewing, doing assignments, etc.

Inquire about **learning resources** available to you. Do you know where the science (engineering, humanities) library is? When are computer labs open? Do you know how the Centre for Student Development can help you deal with academic issues? Before coming to Mac, browse through its internet site. Bookmark the sites that link to learning resources.

'**In university grades drop by 30%**.' Not necessarily. Study regularly (do you know how to study math? Chemistry? Physics? Why not discuss it with your lecturer?). Most probably, you will have to adjust/modify your present study habits. One thing is certain: the amount of work that earned you good marks in high school will not suffice to keep those marks in university.

It is possible to **study hard and still fail a test**. If you fail a test, react immediately. Identify reasons for your poor performance. Visit your professor during her/his office hours, bring your test and discuss it. Be ready to re-examine and modify your study strategies. Do not get discouraged by initial bad marks that you might get.

If you have **problems**, react and deal with it immediately. Ask your professor for advice. Talk to an adviser in your faculty office.

What constitutes **academic dishonesty**? Copying stuff from internet and pasting into your assignment could be considered academic dishonesty. If you are caught, you might fail the course. What other practices are considered academic dishonesty? Be informed about it, so that you don't get in trouble. The course syllabus (for any of your courses) will provide information and links to McMaster policy regarding academic dishonesty.

HOW IS MATH AT UNIVERSITY DIFFERENT FROM HIGH SCHOOL MATH?

Lectures move at a faster pace. Usually, one lecture covers one section from your textbook. Although lectures provide necessary theoretical material, they rarely present sufficient number of worked examples and problems. You have to do those on your own.

Certain topics (trigonometry, exponential and logarithm functions, vectors, matrices, etc.) **will be taught and/or reviewed** in your first-year calculus and linear algebra courses. However, the time spent reviewing in lectures will not suffice to cover all details, or to provide sufficient number of routine exercises – you are expected to do it on your own. Use this manual! Don't leave it at home, bring it with you to McMaster.

You have to know and be proficient with the material covered in the first four chapters of this booklet:

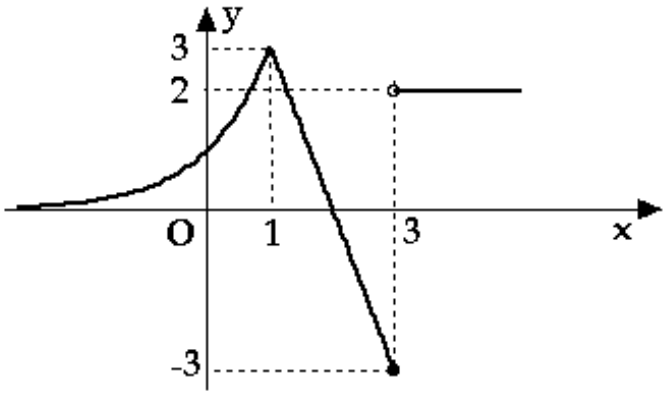
- Basic Algebra
- Basic Formulas from Geometry
- Equations and Inequalities
- Elements of Analytic Geometry.

For instance, computing common denominators, solving equations involving fractions, graphing the parabola $y=x^2$, or solving a quadratic equation will not be reviewed in lectures.

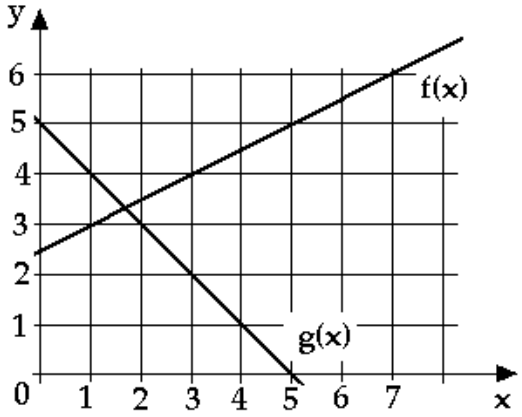
In university, there is **more emphasis on understanding** than on technical aspects. For instance, your math tests and exams will include questions that will ask you to quote a definition, or to explain a theorem, or answer a 'theoretical question.' Here is a sample of questions that appeared on past exams and tests in the first-year calculus course:

- Is it true that $f'(x)=g'(x)$ implies $f(x)=g(x)$? Answering 'yes' or 'no' only will not suffice. You must explain your answer.
- State the definition of a horizontal asymptote.
- Given the graph of $1/x$, explain how to construct the graph of $1+1/(x-2)$.

- Using the definition, compute the derivative of $f(x)=(x-2)^{-1}$.
- Can a polynomial of degree 3 have two inflection points? You must explain your answer to get full credit.
- Given below is the graph of the function f(x). Make a rough sketch of the graph of its derivative f'(x).



- Below are the graphs of two functions, f(x) and g(x). Compute the composition g(f(3)).



You will be allowed - and encouraged - to use your **(graphing) calculator** and/or **computer software** (such as Maple) to study mathematics, to do homework assignments and computer labs. On tests and exams, either no calculator will be allowed (would you really need a calculator to answer any of the above test/exam questions?), or you will be asked to use the calculator that McMaster chose as a standard (this way, everybody uses the same calculator). Calculators and software are an aid, but not a replacement for your brain, and you should treat them as such. If a calculator says something, it is not necessarily a correct answer.

Mathematics is not just formulas, rules and calculations. In university courses, you will study definitions, theorems, and other pieces of 'theory.' Proofs are integral parts of mathematics, and you will meet some in your first-year

courses. You will learn how to approach learning 'theory,' how to think about proofs, how to use theorems, etc.

Layperson-like attitude towards mathematics (and other disciplines!) - accepting facts, formulas, statements, etc. at face value - is no longer acceptable in university. **Thinking (critical thinking!)** must be (and will be) integral part of your student life. In that sense, you must accept the fact that proofs and definitions are as much parts of mathematics as are computations of derivatives and operations with matrices.

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Have you given any thought to **mathematics as a career?** Attend information sessions organized by Mathematics and Statistics Department (will be advertised in lectures), learn about programs and careers in mathematics and statistics. Get informed, keep your options open!

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If, for some reason, you developed negative attitude and feelings towards mathematics in high school, then leave them there! You will have a chance to start fresh at a university. First-year math courses at McMaster start at a level that is appropriate for most high school graduates. Use this manual, read it from cover to cover, get prepared!

LEARNING MATHEMATICS

Learning mathematics requires seriousness, dedication, discipline, concentration, significant amount of time and hard work. Your teachers will help you learn how to learn mathematics.

To learn mathematics means to understand AND to memorize.

To understand something means to be able to correctly and effectively communicate it to somebody else, in writing and orally; to be able to answer questions about it, and to be able to relate it to known mathematics material. Understanding is a result of a thinking process. It is not a mere transfer from the one who understands (your lecturer) to the one who is supposed to understand (you).

How do you make yourself understand math? Ask questions about the material and answer them (either by yourself, or with the help of your colleague, teaching assistant or lecturer). Approach material from various perspectives, study solved problems and work on your own on problems and exercises. Make connections with previously taught material and apply what you just learnt to new situations.

It is necessary to memorize certain mathematics facts, formulas and algorithms. Memorizing is accomplished by exposure: by doing drill exercises, by using formulas and algorithms to solve exercises, by using mathematics facts in solving problems.

The only way to master basic technical and computational skills is to solve a large number of exercises. Drill.

It is impossible to understand new mathematics unless one has mastered (to a certain extent) the required background material.

right approach to learning math:

understand so that you won't have to memorize

wrong approach to learning math:
memorize so that you won't have to understand

think about it!!!

IMPORTANT LITTLE BITS

Come to classes, tutorials and labs regularly, be active! Think, ask questions in class, give feedback to your lecturer.

Lecture by itself will not suffice. You need to spend time on your own doing math: studying, working on assignments, preparing for tests and exams, etc. Rule of thumb: three hours on your own for each hour of lecture.

Plan your study time carefully. Don't underestimate the amount of time you need to prepare for a test, or to work on an assignment; try not to do everything in the last minute (the fact that your hard drive crashed night before your assignment is due is not an acceptable excuse for a late assignment).

Make sure that you are aware of (and use!) **learning resources** available to you. Here are some of them:

- Lectures, tutorials, and review sessions
- Your lecturer's office hours
- Your teaching assistant's office hours
- E-mail and course internet page
- Mathematics help centre in Hamilton Hall
- Thode Library (Science Library)
- Centre for Student Development

Always learn by understanding. Memorizing will not get you too far. Think, do not just read; highlighting every other sentence in your textbook is not studying!

If you are able to explain something to a colleague and answer their questions about it, **then you have learnt it!**

Drill is essential for a success (not just in math!). It's boring, but it works! Solving hundreds of problems will help you gain routine and build confidence you need (together with a few other things) to write good exams.

You will be allowed to use a **(graphing) calculator and/or computer software** for assignments and labs. On tests and exams, you might not be allowed

to use a calculator, or will have to use a model that is accepted as a standard at McMaster. Your instructor will give you detailed information about this.

Note about your lecture notes

Your lecture notes will be your most valuable resource. You will refer to them when you do homework, a computer lab, or prepare for a test or an exam. So:

- during a lecture, take notes
- later, read the notes; make sure that you have correct statements of all definitions, theorems, and other important facts; make sure that all formulas and algorithms are correct, and illustrated by examples
- fill in the gaps in your notes, fix mistakes; supplement with additional examples, if needed
- add your comments, interpret definitions in your own words; restate theorems in your own words and pick exercises that illustrate their use
- write down your questions, and attempts at answering them; discuss your questions with your colleague, lecturer or teaching assistant, write down the answers
- it is a waste of time to try again and yet again to understand a concept; so, once you understood it, write it down correctly, in a way that you will be able to understand later; this way, studying for an exam consists of re-calling and not re-learning; re-calling takes less time, and is easier than re-learning
- keep your notes for future reference: you might need to recall a formula, an algorithm or a definition in another mathematics course.