Math 3C03
M. Min-Oo
Assignment #3

Due: Thursday, October 24th, 2013 in class at the beginning of the lecture

1. Do problem 16.14 on page 552 in the textbook.

2. Expand $f(x) = x^2(1-x)^2$ in terms of Legendre polynomials and verify Parseval's identity.

3. Do problem 18.6 on page 642 in the textbook

4. Do problem 18.10 on page 643 in the textbook

5. Compute the integral:

$$I = \int_{-\infty}^{+\infty} x^2 e^{-x^2} \left(H_n(x)\right)^2 dx$$

(the average potential energy of a quantum harmonic oscillator is related to I)

6. (*bonus question*) Sturm's comparison theorem.
Let $y_1(x)$ and $y_2(x)$ be (non-trivial) solutions of the equations:
$y'' + q_1(x)\, y = 0$ and $y'' + q_2(x)\, y = 0$ respectively on an interval $I = [a, b]$, and assume that $q_1(x) < q_2(x)$ for all $x \in I$. Show that between any two consecutive zeros of y_1, there exists a zero of y_2. *(i.e., the zeros are interlaced).*