Math 3D03
M. Min-Oo
Assignment #2

Due: Tuesday, February 3rd, 2015 in class (at the beginning of the lecture period)

Note: You can use symbolic software **only** *to check your answers (for the integrals for example) but you are required to show your calculations*

1. Evaluate the following definite (real-valued) integrals:

$$(i)\quad \int_0^{2\pi} (\sin\theta)^n \, d\theta \qquad \text{for } n \in \mathbb{N}. \qquad \text{What happens when } n \to \infty\,?$$

$$(ii)\quad \int_{-\infty}^{\infty} \frac{e^{ax}}{1+e^x}\, dx \qquad \text{for } \ 0 < a < 1$$

$$(iii)\quad \int_0^{\infty} \frac{dx}{1+x^n} \qquad \text{where } \ n \geq 2 \ \text{ is an integer}$$

2. Do problems 24.20 and 24.21 on page 869 in the text book.

3. Sum the following infinite series:

$$(a)\quad \sum_{n=1}^{\infty} \frac{1}{n^2+9} \qquad\qquad (b)\quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4} \qquad\qquad (c)\quad \sum_{n=-\infty}^{\infty} \frac{n^2}{n^4-\pi^4}$$

4. How many zeros of the polynomial $z^4 - 5z + 1$ lie in the annulus $1 \leq |z| \leq 2$?

5.

(i) Suppose that $f(z)$ is a non-constant analytic function defined for all $z \in \mathbb{C}$. Show that for every $R > 0$ and for every $M > 0$ there exists a z such that $|z| > R$ and $|f(z)| > M$.

(ii) Suppose that $f(z)$ is a non-constant polynomial. Show that for every $M > 0$ there exists an $R > 0$, such that $|f(z)| > M$ for all $|z| > R$.

(iii) Show that there exists an $M > 0$, such that for every $R > 0$, there exists a z satisfying $|z| > R$ and $|e^z| \leq M$.