Math 3D03
Assignment #3

DUE: THURSDAY, FEBRUARY 26TH, 2015 IN CLASS

Note: You can use symbolic software to check your answers (for the integrals for example) but you are required to show your calculations

1. Show that

$$w = \tan(z)$$

maps the vertical strip $|x| < \frac{\pi}{4}$ in the z-plane onto the unit disk $|w| < 1$ in the w-plane.

2. The complex potential

$$\Omega(z) = z + \frac{1}{z} - i\,\kappa \log(z)$$

where κ is a positive real number, describes a fluid flow around a cylinder with circulation. Locate the stagnation points (as a function of κ) and sketch the streamlines of the flow, using computer software such as Matlab, for the following κ values: $\kappa = 0.5,\ 1.5,\ 2,\ 3$.

3. Find the inverse Laplace transform of

$$\frac{\cosh(x\, s^{\frac{1}{2}})}{s^{\frac{1}{2}} \sinh(a\, s^{\frac{1}{2}})}$$

using a Bromwich contour integral.

4. Show that the Airy function:

$$\psi(z) = Ai(z) = \int_{-\infty}^{\infty} e^{i\left(\frac{1}{3}s^3 + zs\right)}\, ds$$

satisfies Stokes' equation:

$$\frac{d^2\psi}{dz^2} - z\psi = 0$$

and apply the WKB approximation to obtain the following asymptotic expression for the Airy function as $x \to -\infty$ (x real)

$$Ai(x) \approx \frac{1}{\sqrt{2\pi}} x^{-\frac{1}{4}} \sin\left(\frac{2}{3}|x|^{\frac{3}{2}} + \frac{\pi}{4}\right)$$