Math 3D03
Assignment #5

DUE: TUESDAY, MARCH 31ST, 2015 IN CLASS

Note: You can use symbolic software to check your answers but you are required to show your calculations

1. If X_1, X_2, X_3 are independent and identically distributed exponential random variables with the same parameter $\lambda > 0$, compute the probability

$$\mathbb{P}\{max(X_1 + X_2\,,\, X_3) \leq 2\}$$

2. Let X be a random variable having the χ^2 distribution with n degrees of freedom and let Z be a standard normal variate. Assuming X and Z are independent, compute the probability density function of

$$T = \frac{Z}{\sqrt{\frac{X}{n}}}$$

3. For a positive parameter α, the Rayleigh distribution is defined by

$$p(x;\alpha) = \frac{x}{\alpha}\, e^{-\frac{x^2}{2\alpha}} \qquad \text{for } x \geq 0$$

and 0 otherwise.

(i) Compute the mean .

(ii) Given a sample $x_1, \ldots, x_N$, compute the maximum likelihood estimator $\widehat{\alpha}$.

(iii) Show that the log-likelihood function has a local maximum at $\widehat{\alpha}$.

4. Do problem 31.4 on page 1298 in the textbook.

5. Do problem 31.15 on page 1301 in the textbook.

6. (*bonus question*) Let r (*the interest rate*), σ (*the volatility*), K (*strike price*) and T (*time to expiration*) be positive constants. Suppose that the price of a stock at time t can be expressed as

$$S(t) = S(0)\exp\left((r - \frac{1}{2}\sigma^2)t + \sigma\sqrt{t}\,Z\right)$$

where $S(0)$ (*present value of the stock*) is a positive number and Z is the standard normal.

Show that (*Black-Scholes formula for a simple European Call Option*):

$$e^{-rT}\,\mathbb{E}[(S(T) - K)^+] = S(0)\,\Phi(\omega) - e^{-rT}\,K\,\Phi(\omega - \sigma\sqrt{T})$$

where $x^+ = max(x, 0) = \frac{1}{2}(|x| + x)$ for any real number x.