BERNOULLI TRIALS

WIN or LOSE

$$0 \le p \le 1 \qquad q = 1 - p$$

$$p(0) = q \qquad p(1) = p$$

$$\mu = p \qquad \sigma^2 = pq$$

BINOMIAL

Bin(n,p)

sum of n independent Bernoulli trials

sampling **with** replacement

n a positive integer $\qquad 0 \le p \le 1 \qquad q = 1 - p$

$$k = 0, 1, \ldots, n$$

$$p(k) = \binom{n}{k} p^k q^{n-k}$$

$$\mu = np \qquad \sigma^2 = npq$$

probability of k *wins*

HYPERGEOMETRIC

sampling **without** replacement

$$k = 0, 1, \ldots, n$$

$$p(k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$$

$$0 \leq K \leq N, \quad 0 \leq n \leq N, \quad 0 \leq k \leq K, \quad 0 \leq n - k \leq N - K$$

$$\mu = np \qquad \sigma^2 = npq\left(1 - \frac{n-1}{N-1}\right) \qquad \text{where } p = \frac{K}{N}$$

GEOMETRIC

waiting time for first win in a sequence of independent Bernoulli trials

$$k = 1, 2, 3, \ldots$$

$$p(k) = q^{k-1} p$$

$$\mu = \frac{1}{p} \qquad \sigma^2 = \frac{q}{p^2}$$

NEGATIVE BINOMIAL

waiting time for r-th win

r a fixed positive integer

$$k = r,\, r + 1, \ldots$$

$$p(k) = \binom{k-1}{r-1} q^{k-r} p^r$$

$$\mu = r\,\frac{1}{p} \qquad \sigma^2 = r\,\frac{q}{p^2}$$

POISSON

λ a fixed positive real number

$$k = 0, 1, 2, 3, \ldots$$

$$p(k) = e^{-\lambda} \, \frac{\lambda^k}{k!}$$

$$\mu = \lambda \qquad \sigma^2 = \lambda$$

UNIFORM

(rather boring!)

$$a \leq x \leq b$$

pdf:

$$f(x) = \frac{1}{b-a} \qquad \text{for } a \leq x \leq b$$

and

$$f(x) = 0 \qquad \text{otherwise}$$

$$\text{mean} = \frac{a+b}{2} \qquad \text{variance} = \frac{(b-a)^2}{12}$$

STANDARD NORMAL
$\mathcal{N}(0,1)$
mother of all distributions

$$pdf: \qquad f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \qquad z \in \mathbb{R}$$

mean $= 0$ variance $= 1$

NORMAL or GAUSSIAN
$\mathcal{N}(\mu,\sigma)$
shift and scale Z

$$pdf: \qquad f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} \qquad x \in \mathbb{R}$$

mean $= \mu$ variance $= \sigma^2$

EXPONENTIAL

waiting time for the first Poisson event with rate λ

$$f(t) = \lambda\, e^{-\lambda t} \qquad\qquad t \geq 0$$

and

$$f(t) = 0 \qquad \text{otherwise}$$

$$\mu = \frac{1}{\lambda} \qquad \text{variance} = \frac{1}{\lambda^2}$$

ERLANG

waiting time for r-th Poisson event with rate λ

a special case of the Gamma distribution

$$f(t) = \frac{1}{(r-1)!}\lambda\, e^{-\lambda t}(\lambda t)^{r-1} \qquad\qquad t \geq 0$$

and

$$f(t) = 0 \qquad \text{otherwise}$$

$$\text{mean} = \frac{r}{\lambda} \qquad \text{variance} = \frac{r}{\lambda^2}$$

GAMMA

waiting time for r-th Poisson event with rate λ

r doesn't have to be an integer!

$$f(t) = \frac{1}{\Gamma(r)} \lambda\, e^{-\lambda t}\, (\lambda t)^{r-1} \qquad\qquad t \geq 0$$

and

$$f(t) = 0 \qquad \text{otherwise}$$

where the normalizing constant $\Gamma(r)$ is defined to be the integral

$$\Gamma(r) = \int_0^\infty e^{-t}\, t^{r-1}\, dt \qquad r > 0$$

For a positive integer n, $\Gamma(n) = (n-1)!$

$$\text{mean} = \frac{r}{\lambda} \qquad \text{variance} = \frac{r}{\lambda^2}$$

CHI SQUARE

special case of Gamma where $r = \frac{d}{2}$ and $\lambda = \frac{1}{2}$

$d =$ degrees of freedom

sum of squares of d independent standard normals

$$f(t) = \frac{1}{2^{\frac{d}{2}} \Gamma(\frac{d}{2})}\, e^{-\frac{1}{2}t}\, t^{\frac{d}{2}-1} \qquad\qquad t \geq 0$$

and

$$f(t) = 0 \qquad \text{otherwise}$$

$$\text{mean} = d \qquad \text{variance} = 2d$$

STUDENT'S T

d = degrees of freedom $d > 2$

$$pdf: \qquad f(x) = C(d)\left(1 + \frac{x}{d}\right)^{-\frac{d+1}{2}} \qquad x \in \mathbb{R}$$

where the normalizing constant is given by

$C(d) = \frac{1}{\pi\sqrt{d}} \frac{(d-1)(d-3)\cdots 4\cdot 2}{(d-2)(d-4)\cdots 3\cdot 1}$ for odd d

$C(d) = \frac{1}{2\sqrt{d}} \frac{(d-1)(d-3)\cdots 5\cdot 3}{(d-2)(d-4)\cdots 4\cdot 2}$ for even d

$$\text{mean} = 0 \qquad \text{variance} = \frac{d}{d-2}$$