

Math 3Z03
Assignment #3

DUE: MONDAY, FEBRUARY 23RD, 2015

SOLVE ANY 5 OF THE FOLLOWING 6 PROBLEMS:

1. Take any triangle and construct equilateral triangles on its faces. then the centres of these 3 new triangles form another equilateral triangle. Prove it!
(*Napoleon Bonaparte* is supposed to have discovered the above theorem. He had some understanding of mathematics, so it is possibly true)
2. Prove the following formula for the area of a cyclic quadrilateral discovered by Brahmagupta (circa 630 A.D.): $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$, where a, b, c, d are the lengths of the four sides and $s = \frac{1}{2}(a + b + c + d)$.
3. If $F_0 = 1, F_1 = 1, F_2 = 2, F_3 = 3, F_4 = 5, F_5 = 8, \dots$ are the Fibonnaci numbers, show that

$$F_n = \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n-k}{k}$$

for $n > 1$, where $\binom{n}{m}$ denote the binomial coefficients and $\lfloor \frac{n}{2} \rfloor$ is the integral part of $\frac{n}{2}$.

- 4.(i) State the theorem that Blaise Pascal (1623-62) discovered about six points on a conic section (he named it the *mystic hexagram*), that generalizes a theorem by Pappus.
(ii) Make your own drawing of this theorem for the case of an ellipse and show, on the drawing, that the theorem is true.
(iii) Prove the theorem in special case when the conic section is a circle.
5. Solve the following two problems in probability theory that Pascal solved:
(i) How many times must you throw two fair dice in order to have at least a fifty percent chance of getting a double six?
(ii) Alice and Bob are flipping a fair coin. Every time a head comes up, Alice gets 1 point and every time a tail comes up, Bob gets 1 point. The first player to get 10 points wins. Suppose Alice has $10 - m$ points and Bob has $10 - n$ points right now. What is the probability $p(m, n)$ that Alice will win?
6. Prove the following famous product formula for π due to John Wallis (1616-1703):

$$\lim_{n \rightarrow \infty} \frac{2 \cdot 2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot \dots \cdot 2n \cdot 2n}{1 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot \dots \cdot (2n-1) \cdot (2n+1)} = \frac{\pi}{2}$$

(*Hint:* Use integration by parts to compute (and compare) the integrals $\int_0^{\frac{\pi}{2}} \sin^n x \, dx$ for various values of n . This can be found, for example, in J. Stewart's Calculus text book!)