

Math 3Z03
Assignment #5

DUE: MONDAY, MARCH 23RD IN CLASS

SOLVE ANY 5 OF THE FOLLOWING 6 PROBLEMS:

1. Show that if $a^n - 1$ is a prime, then $a = 2$ and n is a prime.
2. A *palindrome* is a number that reads the same backward as forward, such as 2662 or 9,351,539. Prove that any **six digit** palindrome is divisible by 11.
3. What are the five Platonic solids and why are there only five? Compute the radius of the sphere passing through all the vertices of a dodecahedron of side length 1.
4. Prove the following result of *Cauchy*: Let $a_1 \geq a_2 \geq a_3 \geq \dots$ be a non-increasing sequence of positive real numbers.

Show that the series $a_1 + a_2 + \dots + a_n + \dots$ converges if and only if $a_1 + 2a_2 + 4a_4 + 8a_8 + \dots + 2^n a_{2^n} + \dots$ converges.

5. In 1969, *Newton* quit his job as a professor, to work as the Master of Mint. However, he did not entirely give up Mathematics. At about 1772, he posed the following problem:

Suppose that grass grows at a constant rate. For $i = 1, 2, 3$, suppose that it takes x_i cows t_i days to eat all the grass on a_i acres. Prove that:

$$a_1 a_2 x_3 t_3 (t_2 - t_1) + a_2 a_3 x_1 t_1 (t_3 - t_2) + a_3 a_1 x_2 t_2 (t_1 - t_3) = 0$$

6. Show that

$$\int_0^1 x^{-x} dx = \sum_{n=1}^{\infty} n^{-n}$$

(*Euler* knew that)