Math 3GP3
Assignment #1

DUE: TUESDAY, SEPTEMBER 24TH , 2013
Please hand it to me at the beginning of the lecture period in class

1. State and compare the Law of Cosines for Triangles in the three Classical Geometries (Spherical, Euclidean and Hyperbolic) and prove it in the hyperbolic case.

2. Show that the map $w \mapsto z = \frac{w-i}{w+i}$ maps the upper half-plane bijectively onto the unit disk in $\mathbb{C}$ and compute the hyperbolic metric on the unit disk by making the correct change of variables to the metric $ds^2 = \frac{du^2+dv^2}{y^2} = \frac{dwd\bar{w}}{Im(w)^2}$ in the upper half-plane ($w = u + iv$).

3. (i) Compute $L(r)$ = length of a circle of radius r in hyperbolic space?
(ii) Compute $A(r)$ = the area of a disk of radius r in hyperbolic space?
(iii) What happens to $\frac{L(r)^2}{A(r)}$ as $r \to \infty$. Compare with Euclidean space.

4. Show that the area A of a geodesic triangle in hyperbolic space of constant sectional curvature $K \equiv -1$ is given by $A = \pi - (\alpha + \beta + \gamma)$ where α, β, γ are the interior angles.

5. State and prove the addition law for velocities in Special Relativity. Is there a Cauchy-Schwarz inequality for the Lorentzian metric in Minkowski space? Is the triangle inequality true in Minkowski space? What is the relationship between the "twin paradox" and the triangle inequality?

6. (*Bonus Question*) A particle travels (in $\mathbb{R}^n$) from a point P to a point Q at a distance D from P in time T starting with zero velocity at P and ending with zero velocity at Q. Assuming that the trajectory is a smooth curve, show that the magnitude of the acceleration/deceleration has to exceed $\frac{4D}{T^2}$ at some point along the trajectory.