

Math 4B03
Assignment #2

DUE: MONDAY, OCTOBER 6TH, 2014

1. Let

$$\alpha = \frac{y \, dx - x \, dy}{x^2 + y^2} \quad \text{and} \quad \beta = e^z \, dz$$

be 1-forms defined on the complement of the z -axis in \mathbb{R}^3 . Let $\Sigma = S^1 \times [0, 1] \subset \mathbb{R}^3$ be the cylinder parametrized as $(\cos \theta, \sin \theta, t)$, where $\theta \in S^1$ and $t \in [0, 1]$. Compute $\int_{\Sigma} \alpha \wedge \beta$.

2. Find a smooth vector field on S^2 with exactly one zero.
3. Show that $SO(3)$ is diffeomorphic to $\mathbb{R}P^3$
4. Show that an even dimensional sphere S^{2n} does not admit any smooth vector field which is non-zero everywhere.
5. Compute the deRham cohomology groups of T^2 and S^2 and show that every differentiable map $\phi : S^2 \rightarrow T^2$ has degree zero.
6. (bonus question) Compute the total volume V_n of the unit sphere $S^n = \{x \mid |x|^2 = 1\} \subset \mathbb{R}^{n+1}$. What happens as $n \rightarrow \infty$?

Hint: use Gamma functions and Stirling's formula