# Math 4B03
## Assignment #3

Due: Monday, October 20th, 2014

1. Prove the Brouwer fixed point theorem that every differentiable map from the closed unit disk in $\mathbb{R}^n$ to itself has a fixed point.

2. Let $M$ be a compact orientable differential manifold **without boundary**. Show that $M$ is not contractible to a point *if the dimension of $M$ is not zero*

3. The Hopf fibration is defined by

$$\phi : S^3 \to S^2 \qquad q \mapsto \phi(q) = q\, i\, \bar{q}$$

where we view $S^3$ as the set of unit quaternions and $S^2$ as the set of unit imaginary quaternions.

(i) Show that the fibres (i.e. the inverse images of points $\phi^{-1}(z)$) are great circles.

(ii) What is the linking number between any two such fibres?

(iii) Let $\omega$ be the normalized volume form on $S^2$ so that $\int_{S^2} \omega = 1$ Find a 1-form $\beta$ on $S^3$ so that $d\beta = \phi^*(\omega)$ and compute $\int_{S^3} \beta \wedge \phi^*(\omega)$.

(iv) Show that $\phi$ is homotopically non-trivial (i.e., cannot be deformed to the trivial map)

4. (*bonus question*) Compute the deRham cohomology of $\mathbb{C}P^n$.