Math 4B03
Assignment #5

DUE: WEDNESDAY, NOVEMBER 26TH, 2014

1. Prove that even dimensional spheres *of positive dimension* cannot be Lie groups.

2. The metric $ds^2 = -dt^2 + dz^2 + dr^2 + r^2(1-4\mu)^2 d\phi^2$ represents a simple model of a straight infinite cosmic string lying along the z-axis, of mass μ per unit length. Show that in the (r, ϕ)-plane, the 2-dimensional metric $dr^2 + r^2(1-4\mu)^2 d\phi^2$ is isometric to the surface of a cone of semi-angle α, with $\sin\alpha = 1 - 4\mu$.

3. Let $\mathbb{C}P^n$ denote the projective space of all complex lines in $\mathbb{C}^{n+1}$. Let $L \subset \mathbb{C}P^n \times \mathbb{C}^{n+1}$ be defined by $L = \{(l, z) \mid z \in l\}$.

(i) Show that L is a complex line bundle over $\mathbb{C}P^n$

(ii) Find an $n-$ dimensional complex vector bundle $L^\perp$ such that the sum $L \oplus L^\perp$ is a trivial bundle over $\mathbb{C}P^n$

(iii) Show that the tangent bundle T of $\mathbb{C}P^n$ is isomorphic to $Hom(L, L^\perp)$

(iv) Show that $T \oplus \mathbf{1}$, where $\mathbf{1}$ is the trivial line bundle, is isomorphic to $(n+1)\,L$ (sum of $n+1$ copies of L)

(v) For $n = 1$, explain how L is related to the Hopf fibration from the third assignment.

(vi) Compute the first Chern class of the tangent bundle of $\mathbb{C}P^n$

4. (*bonus question*) Compute the deRham cohomolgy groups of $SU(3)$