Summary of Special Functions

	Hermite	Chebyshev
ODE	$y'' - 2xy' + 2\nu y = 0$ $x \in (-\infty, \infty)$	$(1-x^2)y'' - xy' + \nu^2 y = 0$ $x \in [-1, 1]$
Sturm-Liouville Form	$[e^{-x^2} y']' + 2\nu e^{-x^2} y = 0$	$[(1-x^2)^{1/2} y']' + \nu(1-x^2)^{-1/2} y = 0$
Polynomial Solutions	$H_n(x),\ \nu = n$ $n = 0, 1, 2, \ldots$	$T_n(x),\ \nu = n$ $n = 0, 1, 2, \ldots$
Rodrigues' Formula	$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}(e^{-x^2})$	$T_n(x) = \frac{(-1)^n \sqrt{\pi}(1-x^2)^{1/2}}{2^n (n-1/2)!} \frac{d^n}{dx^n}(1-x^2)^{n-1/2}$
Normalization Constant	$\int_{-\infty}^{\infty} H_n(x) H_n(x) e^{-x^2}\, dx = 2^n n! \sqrt{\pi}$	$\int_{-1}^{1} T_n(x) T_n(x) (1-x^2)^{-1/2}\, dx = \pi/2,\ n \geq 1$
Generating Function	$e^{2xu-u^2} = \sum_{n=0}^{\infty} \frac{H_n(x)}{n!} u^n$	$\frac{1-xu}{1-2xu+u^2} = \sum_{n=0}^{\infty} T_n(x) u^n$
Recurrence Relations	$H_{n+1} = 2xH_n - 2nH_{n-1}$ $H_n' = 2nH_{n-1}$	$T_{n+1} - 2xT_n + T_{n-1} = 0$

	Legendre	Associated Legendre
ODE	$(1-x^2)y'' - 2xy' + \lambda y = 0$ $x \in (-1,1)$	$(1-x^2)y'' - 2xy' + \left[\lambda - \frac{m^2}{1-x^2}\right] y = 0$ $x \in (-1,1)$
Sturm-Liouville Form	$[(1-x^2)y']' + \lambda y = 0$	$[(1-x^2)y']' - \frac{m}{1-x^2} y + \lambda y = 0$
Polynomial Solutions	$P_\ell(x),\ \lambda = \ell(\ell+1)$ $\ell = 0, 1, 2, \ldots$	$P_\ell^m(x) = (1-x^2)^{m/2} \dfrac{d^m P_\ell(x)}{dx^m}$
Rodrigues' Formula	$P_\ell(x) = \dfrac{1}{2^\ell \ell!} \dfrac{d^\ell}{dx^\ell}(x^2-1)^\ell$	$P_\ell^m(x) = \dfrac{1}{2^\ell \ell!}(1-x^2)^{m/2} \dfrac{d^{\ell+m}}{dx^{\ell+m}}(x^2-1)^\ell$
Normalization Constant	$\displaystyle\int_{-1}^{1} P_\ell(x) P_\ell(x)\, dx = \frac{2}{2\ell+1}$	$\displaystyle\int_{-1}^{1} P_\ell^m(x) P_\ell^m(x)\, dx = \frac{2}{2\ell+1} \frac{(\ell+m)!}{(\ell-m)!}$
Generating Function	$\displaystyle\frac{1}{\sqrt{1-2xu+u^2}} = \sum_{n=0}^{\infty} P_n(x) u^n$	$\displaystyle\frac{(2m)!(1-x^2)^{m/2}}{2^m m!(1-2xu+u^2)^{m+1/2}} = \sum_{n=0}^{\infty} P_n^m + m(x) u^n$
Recurrence Relations	$P'_{\ell+1} = P_\ell + 2xP'_\ell - P'_{\ell-1}$ $P'_{\ell+1} = (\ell+1)P_\ell + xP'_\ell$ $P'_{\ell-1} = -\ell P_\ell + xP'_\ell$	$P_n^{m+1} = \dfrac{2mx}{\sqrt{1-x^2}} P_n^m + [m(m-1) - n(n+1)] P_n^{m-1}$ $P_n^m = \dfrac{P_{n+1}^{m+1} - P_{n-1}^{m+1}}{(2n+1)\sqrt{1-x^2}}$

	Laguerre	Associated Laguerre
ODE	$xy'' + (1-x)y' + \nu y = 0$ $x \in [0, \infty)$	$xy'' + (m+1-x)y' + \nu y = 0$ $x \in [0, \infty)$
Sturm-Liouville Form	$[xe^{-x}y']' + \nu e^{-x} y = 0$	$[x^{m+1}e^{-x}y']' + \nu x^m e^{-x} y = 0$
Polynomial Solutions	$L_n(x),\ \nu = n$ $n = 0, 1, 2, \ldots$	$L_n^m(x) = (-1)^m \frac{d^m}{dx^m} L_{n+m}(x)$
Rodrigues' Formula	$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n}(x^n e^{-x})$	$L_n^m(x) = \frac{e^x x^{-m}}{n!} \frac{d^n}{dx^n}(x^{n+m} e^{-x})$
Normalization Constant	$\int_0^\infty L_n(x) L_n(x) e^{-x}\, dx = 1$	$\int_0^\infty L_n^m(x) L_n^m(x) x^m e^{-x}\, dx = \frac{(n+m)!}{n!}$
Generating Function	$\frac{e^{-xu/(1-u)}}{1-u} = \sum_{n=0}^\infty L_n(x) u^n$	$\frac{e^{-xu/(1-u)}}{(1-u)^{m+1}} = \sum_{n=0}^\infty L_n^m(x) u^n$
Recurrence Relations	$L_{n+1} = \frac{f(x;2n)}{(n+1)} L_n - \frac{n}{(n+1)} L_{n-1}$ $xL_n' = nL_n - nL_{n-1}$ $f(t;k) = k+1-t$	$L_{n+1}^m = \frac{f(x;2n+m)}{(n+1)} L_n^m - \frac{n+m}{(n+1)} L_{n-1}^m$ $x(L_n^m)' = nL_n^m - (n+m)L_{n-1}^m$ $f(t;k) = k+1-t$