2004-11-04

STATS 3N03 / 3J04 TEST 2/2b SOLNS

1) Siméon Denis Poisson (1)

2) Let X = Number of passengers who show up
Assume: Passengers show up or not, independently of each other. (1)
$\Rightarrow X \sim Bin(125, 0.9)$

$P(\text{At least 1 empty seat}) = P(X<120)$ (5)

$$= 1 - \sum_{x=120}^{125} \binom{125}{x} .9^{x} .1^{125-x} = 0.9885678$$

NORMAL APPROX. (WITHOUT c.c.): $\Phi\left(\frac{119-112.5}{\sqrt{11.25}}\right) = \Phi(1.94) = 0.974$

(WITH c.c.): $\Phi\left(\frac{119.5-112.5}{\sqrt{11.25}}\right) = \Phi(2.09) = 0.982$

TEST 2b: $P(\text{At least 2 empty seats}) = P(X \le 118)$

$$= 1 - \sum_{x=119}^{125} \binom{125}{x} .9^{x} .1^{125-x} = 0.9717376$$

(WITHOUT cc): $\Phi\left(\frac{118-112.5}{\sqrt{11.25}}\right) = \Phi(1.64) = 0.949$

(WITH cc) $\Phi\left(\frac{118.5-112.5}{\sqrt{11.25}}\right) = \Phi(1.79) = 0.963$

3) Let X = Waiting time for a randomly chosen customer.

Assume (1) Waiting times are independent [a dubious assumption if they line up for sequential service] and (2) the central limit theorem applies. (2)

Given: $\mu_X = 8.2$ min, $\sigma_X = 1.5$ min.

Hence $\bar{X} \sim AN\left(8.2, \frac{1.5^2}{49}\right)$ so

$$P(5 < \bar{X} < 10) \approx \Phi\left(\frac{10-8.2}{\frac{1.5}{7}}\right) - \Phi\left(\frac{5-8.2}{\frac{1.5}{7}}\right)$$

$$= \Phi(8.4) - \Phi(-14.93) = 1 \qquad 5$$

Test 2b: $P(8 < \bar{X} < 9) \approx \Phi\left(\frac{9-8.2}{\frac{1.5}{7}}\right) - \Phi\left(\frac{8-8.2}{\frac{1.5}{7}}\right)$

$$= \Phi(3.73) - \Phi(-0.93) = 0.999904 - 0.176185 = 0.824$$

4) Let X = Number of failures in an 8hr. shift

Given: $E(X) = (0.02)(8) = 0.16$

Assume: (1) Failures one at a time, (2) independent of each other 2

$\Rightarrow X \sim Pois(0.16)$

$$P(X=0) = e^{-0.16} = 0.8521 \qquad 5$$

Test 2b: $P(X \geq 2) = 1 - P(X \leq 1) = 1 - e^{-.16} - (.16)e^{-.16} = 0.0115$

5) PARAMETER: A scalar or vector that indexes a family of probability distributions. 1

STATISTIC: Any function of the observations in a sample. It may not include any unknown parameters. 1

SAMPLING DISTRIBUTION: The distribution of a statistic. It describes how the statistic will vary from one sample to another. 1