STATISTICS 2N03 / 3J04 - TEST 3a SOLUTIONS

1. (a) SEE DEFINITIONS WEB PAGE.

   (b) YOU ARE COMPARING MEANS OF TWO INDEPENDENT NORMAL POPULATIONS, AND YOU WANT TO CHECK THE ASSUMPTION THAT BOTH HAVE THE SAME VARIANCE.

2. IT DEMONSTRATES THAT THE SAMPLE VARIANCE s^2 IS AN UNBIASED ESTIMATE OF POPULATION VARIANCE σ^2 FOR ANY DISTRIBUTION (PROVIDED THE DATA ARE INDEPENDENT).

3. [PLEASE SEE LAST YEAR'S T03 SOLUTIONS FOR THE MARKING SCHEME.]

(a) TWO INDEPENDENT SAMPLES.

$n_1 = 11 \quad \bar{x}_1 = 8.02 \quad s_1^2 = 0.01248$

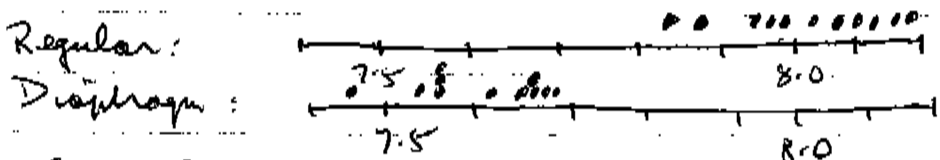
$n_2 = 11 \quad \bar{x}_2 = 7.601\dot{8} \quad s_2^2 = 0.00391636$

TO TEST THE HYPOTHESIS OF NO DIFFERENCE IN VARIANCE:

$$F_0 = \frac{0.01248}{0.0039164} = 3.187 \quad \text{REF: } F(10,10)$$

SINCE $f_{.05,10,10} = 2.98$ AND $f_{.025,10,10} = 3.72$, 2-SIDED $.05 < P < .1$

∴ WE ACCEPT ASSUMPTION OF HOMOSCEDASTICITY.



FROM GRAPH, IT IS OBVIOUS THAT MEANS DIFFER.

EQUAL-VARIANCES t-TEST: $t_0 = \dfrac{8.02 - 7.601\ddot{8}}{.09054\sqrt{\frac{1}{11}+\frac{1}{11}}} = 10.831$

REF: $t(20)$

SINCE $t_{.0005,20} = 4.587$, 2-SIDED $P \ll 0.001$

∴ STRONG EVIDENCE THAT THE MEANS DIFFER.

ASSUME: NORMALITY (OK BY GRAPH), INDEPENDENCE (CAN'T TEST), HOMOSCEDASTICITY (OK BY F TEST).

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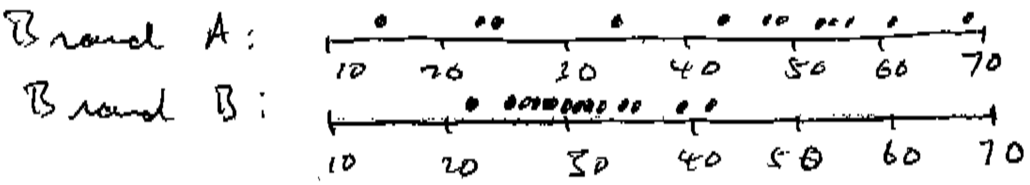
(b) INDEPENDENT SAMPLES

$n_1 = 12 \quad \bar{x}_1 = 43.691\dot{6} \quad s_1^2 = 286.3862878$

$n_2 = 13 \quad \bar{x}_2 = 31 \quad s_2^2 = 31.51\dot{6}$



TO TEST THE HYPOTHESIS OF NO DIFFERENCE IN VARIANCE:

$$F_0 = \frac{286.3862878}{31.516} = 9.087 \quad \text{Ref}: F(11,12)$$

Since $f_{.01,11,12} = 4.2$, $P \ll 0.02$ (2-SIDED)

∴ THERE IS STRONG EVIDENCE ($P \ll 0.02$) THAT THE BRANDS ARE NOT EQUALLY VARIABLE. BRAND A IS MORE CONSISTENT THAN BRAND B.

ASSUMPTIONS: NORMALITY (LOOKS OK ON DOT PLOTS), INDEPENDENCE (CAN'T TEST).

WE COULD DO THE UNEQUAL-VARIANCES t-TEST TO GET $t_0 = 2.475$, $df = 13.2$, $P \simeq 0.02$ OR THE EQUAL-VARIANCES t-TEST TO GET

$$t_0 = \frac{43.691\dot{6} - 31}{12.3659\sqrt{\frac{1}{12}+\frac{1}{13}}} = 2.5597 \quad \text{Ref}: t(23)$$

SINCE $t_{.01,23} = 2.500$, $P \approx 0.02$

EITHER WAY, THERE IS SOME EVIDENCE ($P \approx 0.02$) THAT THE MEAN TIME TO RAISE TEMPERATURE BY 10° IS NOT THE SAME FOR EACH BRAND.

ASSUMPTIONS: SEE F-TEST ASSUMPTION ABOVE.