Wald and Likelihood Ratio Tests for a Gamma Shape Parameter

The following code performs Wald and Likelihood ratio tests for the shape parameter of a gamma distribution. First, the shape parameter is held at the hypothesized value shape0H and the rate is estimated by maximum likelihood scoring. Then the unconstrained MLE of shape and rate is computed by scoring. Finally, the Wald and Likelihood Ratio pivotal quantities are computed, with their corresponding P-values. The methods are illustrated with a sample of 200 observations from a gamma distribution with shape = 2 and rate = 1.

```
> gammatestWL
function (xx, shape0H, rate0)
{
    n <- length(xx)
    xbar <- mean(xx)
    logxbar <- mean(log(xx))
    theta <- rate0
    repeat {
        theta0 <- theta
        rate <- theta0
        S <- n * (shape0H/rate - xbar)
        I <- n * shape0H/rate^2
        theta <- theta0 + S/I
        repeat {
            if (theta > 0)
                break
            theta <- (theta0 + theta)/2
        }
        if (abs(theta - theta0) < 1e-08)
            break
    }
    ratehathat <- theta
    thetahathat <- c(shape0H, ratehathat)
    theta <- c(shape0H, ratehathat)
    repeat {
        theta0 <- theta
        shape <- theta0[1]
        rate <- theta0[2]
        S <- n * matrix(c(log(rate) - digamma(shape) + logxbar,
            shape/rate - xbar), ncol = 1)
        I <- n * matrix(c(trigamma(shape), -1/rate, -1/rate,
            shape/rate^2), ncol = 2)
        theta <- theta0 + solve(I) %*% S
        repeat {
            if (min(theta) > 0)
                break
            theta <- (theta0 + theta)/2
        }
        if (max(abs(theta - theta0)) < 1e-08)
            break
    }
    thetahat <- as.vector(theta)
    ChisqW <- t(thetahathat - thetahat) %*% I %*% (thetahathat -
        thetahat)
    PW <- 1 - pchisq(ChisqW, 1)
    ChisqL <- -2 * (sum(log(dgamma(xx, thetahathat[1], thetahathat[2]))) -
        sum(log(dgamma(xx, thetahat[1], thetahat[2]))))
    PL <- 1 - pchisq(ChisqL, 1)
    list(thetahathat = thetahathat, thetahat = thetahat, ChisqW = ChisqW,
        "P(>ChisqW)" = PW, ChisqL = ChisqL, "P(>ChisqL)" = PL)
}
```

Test the hypothesis that shape = 2

```
> gammatestWL(xx,2,1)
$thetahathat
[1] 2.0000000 0.9870246

$thetahat
[1] 1.769417 0.873229

$ChisqW
         [,1]
[1,] 2.001084

$"P(>ChisqW)"
           [,1]
[1,] 0.1571868

$ChisqL
[1] 1.832668

$"P(>ChisqL)"
[1] 0.1758128
```

Test the hypothesis that shape = 1.5

```
> gammatestWL(xx,1.5,1)
$thetahathat
[1] 1.5000000 0.7402685

$thetahat
[1] 1.769417 0.873229

$ChisqW
        [,1]
[1,] 2.731861

$"P(>ChisqW)"
           [,1]
[1,] 0.09836463

$ChisqL
[1] 3.069230

$"P(>ChisqL)"
[1] 0.07978724
```