

The equivariant cohomology rings of regular nilpotent Hessenberg varieties in Lie type A

Tatsuya Horiguchi

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Let n be a fixed positive integer and $h : \{1, 2, \dots, n\} \rightarrow \{1, 2, \dots, n\}$ a Hessenberg function. The main result is to give a systematic method for producing an explicit presentation by generators and relations of the equivariant and ordinary cohomology rings with \mathbb{Q} coefficients of any regular nilpotent Hessenberg variety in Lie type A . Specifically, we give an explicit algorithm, depending only on the Hessenberg function h , which produces the n defining relations in the equivariant cohomology ring. Our result generalizes known results: for the case $h = (2, 3, 4, \dots, n, n)$, which corresponds to the Peterson variety Pet , we recover the presentation of the equivariant and ordinary cohomology ring of Pet given previously by Fukukawa, Harada, and Masuda. Moreover, in the case $h = (n, n, \dots, n)$, for which the corresponding regular nilpotent Hessenberg variety is the full flag variety $Flags(\mathbb{C}^n)$, we can explicitly relate the generators of our ideal with those in the usual Borel presentation of the cohomology ring of $Flags(\mathbb{C}^n)$. This is a joint work with Hiraku Abe, Megumi Harada, and Mikiya Masuda.