

Math 3F03, Fall 2014
Assignment 1

This assignment is due on **Monday, September 29**, in class at the beginning of lecture. **Late assignments will not be graded.**

You must show your work and justify your assertions carefully in order to receive full credit.

Page references refer to the course textbook, *Differential Equations, Dynamical Systems, and an Introduction to Chaos* by Hirsch, Smale, and Devaney, Third Edition.

- 1.) Page 16, exercise 1.
- 2.) Page 16, exercise 2.
- 3.) Page 17, exercise 5.
- 4.) Page 18, exercise 12.
- 5.) Page 37, exercises 2.
- 6.) Page 37, exercise 3.
- 7.) Page 38, exercise 14.
- 8.) Suppose $\lambda = \alpha + i\beta$ is a complex eigenvalue of a 2×2 matrix A with real entries, having corresponding eigenvector V . Show that the complex conjugate $\bar{\lambda} = \alpha - i\beta$ is also an eigenvalue of A , and that its corresponding eigenvector is \bar{V} , the vector whose entries are the complex conjugates of the entries of V .
- 9.) Suppose a 2×2 matrix A with real entries has a complex eigenvalue $\lambda = \alpha + i\beta$ with corresponding eigenvector V . Then the complex-valued function $e^{\lambda t}V = X(t)$ solves the system $X' = AX$. Write $X(t) = \operatorname{Re}X(t) + i\operatorname{Im}X(t)$, where both $\operatorname{Re}X(t)$ and $\operatorname{Im}X(t)$ are real-valued functions. Show that as functions of t , $\operatorname{Re}X(t)$ and $\operatorname{Im}X(t)$ are linearly independent. (Hint: Start by showing that the real and imaginary parts of V are linearly independent.)