MATH 3F03 Assignment 1
Marking Comments

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Preamble

How to use this document: Check your marked assignment solutions for boxed numbers. Each boxed number corresponds to a comment in this document. If any points are deducted from your solution related to a comment, the number of points deducted is included in brackets after that comment.

For example,

7. *Page 555, exercise 6.*

$\boxed{1}$ Did not check stability of equilibria **(-2)**

means that in question 7 (which is exercise 6 on on page 555 of *Hirsch, Smale, Devaney*) you forgot to check the stability of equilibria, and so I deducted two points.

All questions are marked out of 10.

Note: Only the most commonly applicable comments are included here. Your assignment may have some additional, specific comments.

Comments

1. *Page 16, exercise 1.*

$\boxed{1}$ Did not specify that the general solution of $x(t) = ke^{at} - \frac{3}{a}$ is not valid for $a = 0$ and did not provide a general solution or discuss equilibria for the $a = 0$ case. **(-3)**

$\boxed{2}$ Did specify that the general solution of $x(t) = ke^{at} - \frac{3}{a}$ and/or equilibrium $x^* = -\frac{3}{a}$ is not valid for $a = 0$ but still neglected to provide a general solution or discuss equilibria for the $a = 0$ case. **(-2)**

$\boxed{3}$ Did not specify that the general solution of $x(t) = ke^{at} - \frac{3}{a}$ is not valid for $a = 0$ and did not provide a general solution for the $a = 0$ case. **(-2)**

2. *Page 16, exercise 2.*

Each part of this question was marked out of two points.

$\boxed{1}$ Remember to use filled circles ($\bullet$) to denote sinks (stable solutions) and unfilled circles ($\circ$) to denote sources (unstable solutions) when drawing phase lines.

$\boxed{2}$ Solutions that are neither sources nor sinks can be "semi-stable" – that is, stable on one side of the equilibrium, and unstable on the other side. If that's the case, say it. You should also denote these by half-filled circles, where the (un)filled side is the (un)stable side.

$\boxed{3}$ Remember to include arrows on the phase line to indicate direction of flow.

$\boxed{4}$ In part e, note that for $x' = |1 - x^2| = f(x)$, $f(x)$ is *not* differentiable at the equilibrium points $x^* = \pm 1$ (these are cusps), and so you *cannot* use the second derivative test to check the stability of these points (since $f''(x^*)$ does not exist). Note further that the second derivative test *does not say* that if $f'(x^*)$ does not exist, then x^* is semi-stable. **(-1)**

$\boxed{5}$ In part e, you did not justify how you knew the stability of the equilibria by showing your work. **(–0.5)**

6. *Page 37, exercise 3.*

$\boxed{1}$ While you managed to match both systems with saddles at $X^* = (0,0)$ to their corresponding direction fields (that is, 1. with c. and 3. with d.), your argument was not clear and/or compelling enough to convince me that it shouldn't instead be the case that 1. corresponds to d. and 3. corresponds to c. **(-2)**

$\boxed{2}$ Did not recognize that $\lambda = 0$ corresponds to a line of equilibria. **(-1)**

$\boxed{3}$ Unclear argument. **(-1)**

$\boxed{4}$ You matched all of the direction fields correctly, but you provided no justification for your choices. The assignment instructions explicitly state "You must show your work and justify your assertions carefully in order to receive full credit." Moreover, in tutorial, I emphasized that the most important part of this question is that you provide a clear and compelling argument for why you matched each solution with a particular direction field. **(–4)**.

$\boxed{5}$ You did not manage to distinguish the two systems with saddles at $X^* = (0,0)$. **(-3)**

8. Generally well done.

9. $\boxed{1}$ It is true that if two vectors, v_1 and v_2, are linearly independent, then $c_1v_1 + c_2v_2 = 0$ if and only if $c_1, c_2 = 0$. However, it is not the case that if $c_1v_1 + c_2v_2 = 0$ and $c_1, c_2 = 0$, then v_1 and v_2 are linearly independent. If this second assertion were true, any pair of vectors would be linearly independent! **(-2)**

$\boxed{2}$ Remember that

$$\begin{aligned}\mathrm{Re}(\mathrm{X(t)}) &= e^{\alpha t}\cos(\beta t)\mathrm{Re}(\mathrm{V}) - \mathrm{e}^{\alpha t}\sin(\beta \mathrm{t})\mathrm{Im}(\mathrm{V})\\ \mathrm{Im}(\mathrm{X(t)}) &= e^{\alpha t}\cos(\beta t)\mathrm{Im}(\mathrm{V}) + \mathrm{e}^{\alpha t}\sin(\beta \mathrm{t})\mathrm{Re}(\mathrm{V}).\end{aligned}$$

Even if Re(V) and Im(V) are shown to be linearly independent, Re(X(t)) and Im(X(t)) are not "obviously" linearly independent because their coefficients in the basis of $\{\mathrm{Re}(\mathrm{V}), \mathrm{Im}(\mathrm{V})\}$ are not constants but functions of time...