Math 3F03, Fall 2014 Midterm Exam 1

Name:

Student Number:

Instructions

- No calculators, notes, or books are permitted during this exam.
- This test consists of five questions and 25 points.
- All answers and work must be written in your exam booklet. **Make sure to put your name and student number on your exam booklet before handing it in**.
- For all questions you **must** show your work for full credit.
- You have 50 minutes.
- Good Luck!

1.) (5 pts) The family of differential equations $x' = x^3 - ax$ depends on a parameter a. Sketch the corresponding bifurcation diagram.

2.) (5 pts) Consider the second-order differential equation: $x'' + 3x' + 2x = 0$.
(i) Rewrite this ODE as a system of equations $X' = AX$, making sure to indicate any change of variables you use.
(ii)Find the general solution of the system in (i).
(iii)Find J, the real Jordan canonical form of the matrix A, and give the matrix T for which $T^{-1}AT = J$.
(iv)Sketch a phase portrait of the system in (i). Make sure to label it clearly as a source, sink, saddle, etc.

3.) (5 pts) Show that every solution of $x' = ax$ has the form $x(t) = ke^{at}$ for some $k \in \mathbb{R}$. That is, show that the solution $x = ke^{at}$ is *unique* up to the choice of k.

4.) (5 pts) Let A be a $2X2$ matrix with distinct real distinct eigenvalues λ_1 and λ_2. Prove that the eigenvectors V_1 and V_2 associated to λ_1 and λ_2, respectively, must be linearly independent.

5.) (5 pts) Give a condition under which two $2X2$ hyperbolic matrices A_1 and A_2 will be conjugate to one another, and list the conjugacy classes (i.e. families of mutually conjugate hyperbolic matrices) determined by this condition.